PHILADELPHIA UNIVERSITY
ELECTRICAL ENGINEERING DEPARTMENT

Power System Stability

Course Notes

PART-1

Dr. A.Professor Mohammed Tawfeeq Al-Zuhairi

September 2012

Definition of stability

The stability of a power system is defined as the ability of the power system to return to normal or stable operation after having been subjected to some form of disturbance.

- **Disturbances threatening the stability of a system:**

Small and large disturbances

A power system is subjected to a variety of disturbances. These are classified into two categories:

• Small disturbances (perturbations)

• Large disturbances (perturbations)

Small Perturbations

Perturbations are characterized as small if the changes in system states are small due to these perturbations. Random changes in load which occur in the system continuously are an example of small perturbation. Stability problem associated with small perturbation is known as dynamic stability.

Large perturbations

A power system may be subjected to large perturbations such as:

• Occurrence of faults on the line

• Loss of large generating units

• Loss of major transmission facilities

• Loss of large loads

❖ **Energy fluctuation as a result of disturbance:**

- between capacitors and inductances
- between rotating masses (most important factor)
- boilers in power plants, or combustion chambers

❖ **Changes can be seen in the quantities *P, Q, V, I***

- time constants between 10 ms–10 s
- period of oscillations between 0.5–5 s

Categorization of stability in practice

1- **Rotor angle stability** (Loss of synchronism)
Load angle differences of generators become too large causing tripping of generators

2- **Voltage stability**
Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance. Collapse of voltage makes interconnected operation impossible

3- **Frequency stability**
Frequency stability refers to the ability of a power system to maintain steady frequency at all buses in the system after being subjected to a disturbance. Collapse of frequency makes interconnected operation impossible

Rotor angle stability (Synchronous operation)

The stability of synchronous machines depends on the state of the network and the nature of the disturbance. **In general, the rotor angle stability problems are classified into three types:**

- steady-state stability
- transient stability
- dynamic stability

❖ **Steady state stability:** The ability of the power system to remain in synchronism when subject to small disturbances (Small Perturbations due to random variation of loading).

❖ **Transient stability:** The stability problem associated with large perturbations is known as transient stability. It concerns with the study of behavior of the power system when is subject to fast changes with large magnitudes. Transient stability studies are normally performed for a period of 1 second. The faults that are normally taken into consideration for the transient stability studies are: faults of heavily loaded lines, the tripping of a loaded generator or the dropping of a large load.

- The transient stability limit is generally lower than the steady state stability limit.
- In practice, transient stability is determined by computer calculation. Manual calculation is possible only in simple cases.

❖ **Dynamic stability**: After a fault, an individual synchronous machine may remain in synchronism during the first swing. After this, strong electromechanical oscillations occur may cause the system to have natural oscillations. The system is said to be *dynamically stable* if the oscillations don't exceed a certain magnitude and die out quickly (i.e., the system is *well-damped*). Dynamic stability of a given power system can be improved through the use of power system stabilizers. Dynamic stability study is normally carried out for a period of 5 to 10 second and sometimes to 30 seconds.

Rated faults on the grid

- ❖ a three-phase short circuit of any line, followed by a successful high speed automatic reclosing.
- ❖ a single-phase earth fault of any line, followed by a failed high-speed automatic reclosing.
- ❖ a three-phase short circuit of any bus bar, followed by a disconnection (switching off) of the bus.

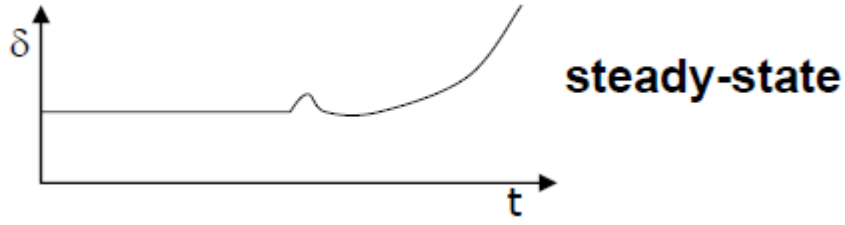
The situation is acceptable, when:

- the synchronism of the network is not lost during the oscillation (swing) followed by the fault.
- sustained (undamping) oscillations do not occur after the fault has been cleared.
- voltages of the 400 kV network are at least 370 kV at all stations when the oscillation following the fault has damped out.
- network components are not exposed to permanent overload.
- single faults do not lead to an expanding disturbance .
- frequency does not fall below the frequency of 48.5 Hz that activates the automatic disconnection of loads.
- frequency does not exceed 51.5 Hz .

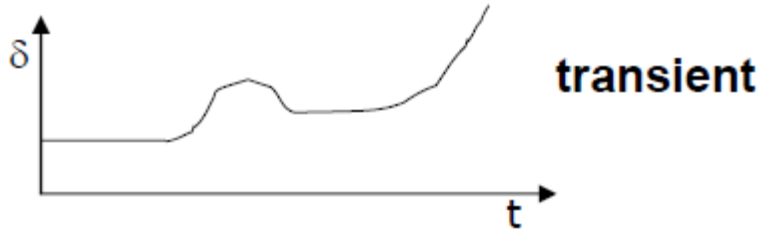
The power system normally has many interconnected generating stations, each with several generators. For the purpose of stability studies machines located at any generating station are normally considered as one large equivalent machine. This also applies for machines connected by low impedance lines. In this way a multi-machine system can be reduced to an equivalent few machine system.
Normally, if the system falls out of step (synchronism lost) machines at any power station stay in synchronism although they go out of step with other machines located far away at other generating stations.

Typical swing curves in the loss of stability

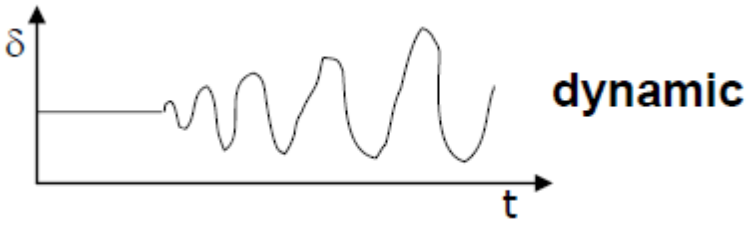
1. At the power transfer limit (steady-state stability), the generator falls out of synchronism as a result of a small transient phenomenon on the network.



2. Transient stability is lost as a result of a fault on the network; the generator accelerates (speeds up) after the loss of synchronism.



3. Dynamic stability is lost as a result of sustained power angle oscillations caused by a fault.

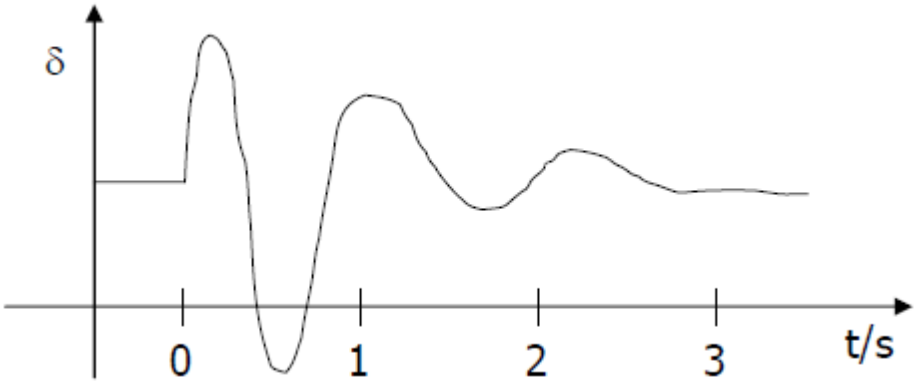


4. Exceeding the limit of voltage stability as a result of a fault leads to an accelerated decrease in voltages and to a voltage collapse.

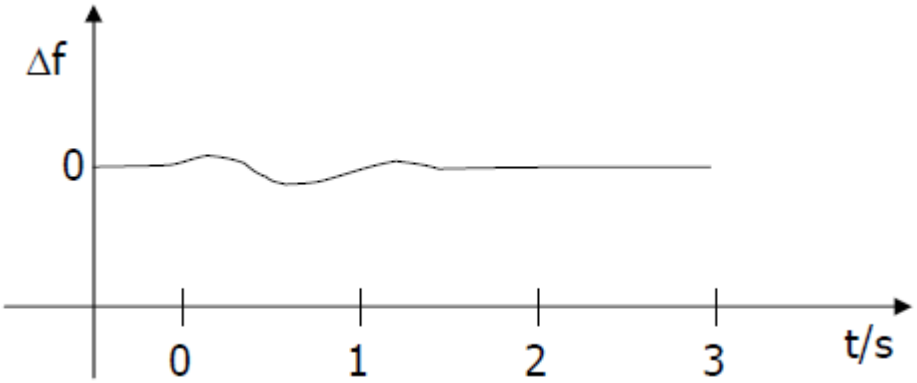
Examples of stability

1. Stable oscillation

Rotor angle swing

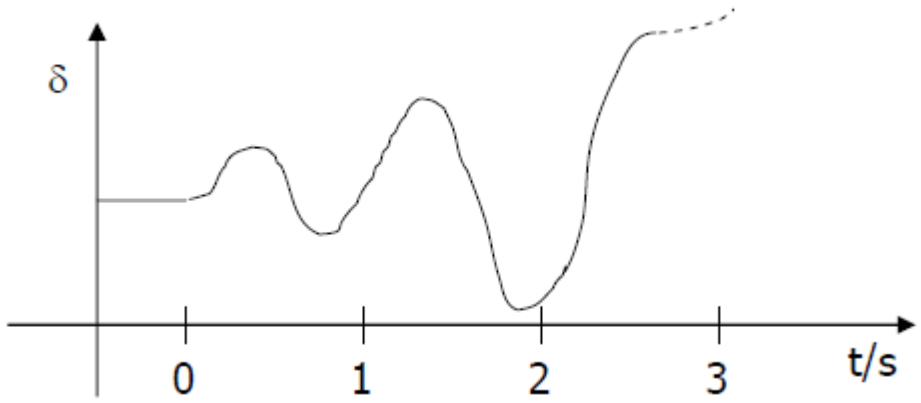


Frequency Variation

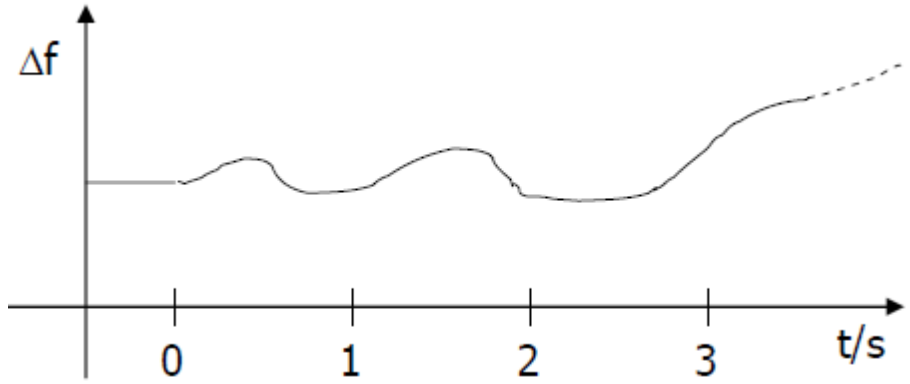


2. Loss of stability (Caused by a severe fault)

Rotor angle swing



Frequency Variation



Lecture 2 : Review

- Power System Basic Concept Review

  The function of an electric power system is to convert energy from one of the naturally available forms (fossil fuel, hydrolic or nuclear, etc) to the electrical form and to transport it to the points of consumption.

- Outline of power system Structure

  Electric power systems vary in size and structural components. However, they all have the same basic characteristics:

  - Comprize of 3-phase ac systems operating at constant voltage.
  - Use synchronous machines for generation of electricity.
  - Transmit power over significant distance to the consumers.

- In general we can summarize the structure of electric power system as follows:

  1. Generation : Use synchronous generators in power plants (generating Station - GS).
  2. Transmission : Transmission networks are classified into the following subsystems:
     * Transmission System 220 kV→.
     * Subtransmission system 132 kV, 66kV

3. Distribution : 33kV, 11kV, 400/230V.

Fig.1 shows a typical power system basic elements.

- Power System Requirements :

A good electric power system should meet the following requirements :

1. The system must be able to meet the continuously changing load demand. Therefore, adequate "spining reserve" should be maintained and appropriately controlled at all times.

2. The system should supply energy at minimum cost.

3. The "quality" of the power supply must meet certain minimum standards with regard to the following factors :-
   (a) Voltage constancy
   (b) Frequency Constancy.
   (c) Good level of reliability.

- Power System Control

In order to meet the above requirements several level of controls involving a complex array of devices are used :

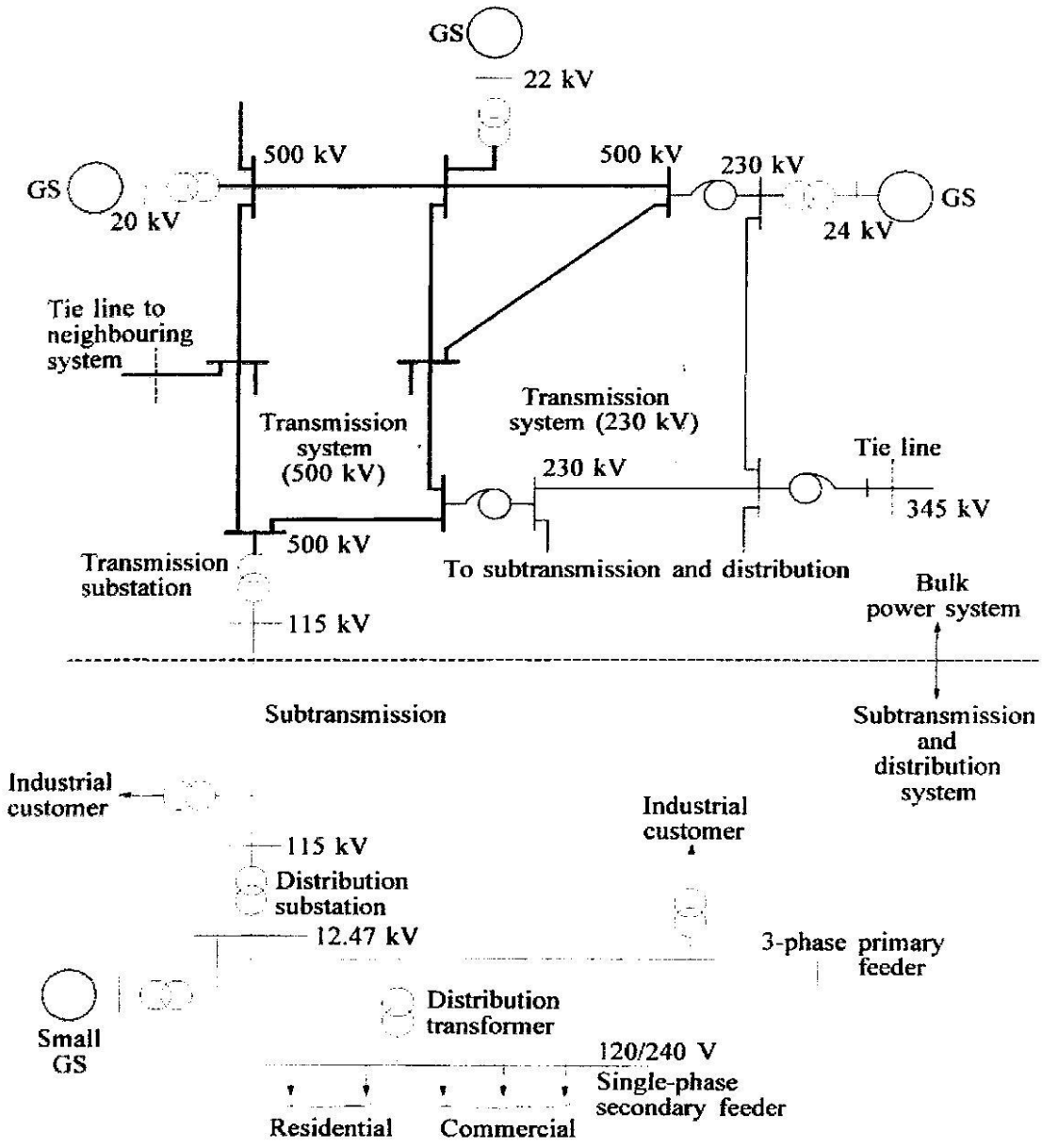


Fig.1 Typical power system structure.

1. In a generating unit : The controls consist of :

- Prime-mover controls :
  - Speed control
  - Input energy control (boiler, temperature, pressure and flow).
- Excitation controls :
  - Regulate generator voltage.
  - Regulate reactive power output(MVARs).

Fig.2 shows typical generating unit control systems.

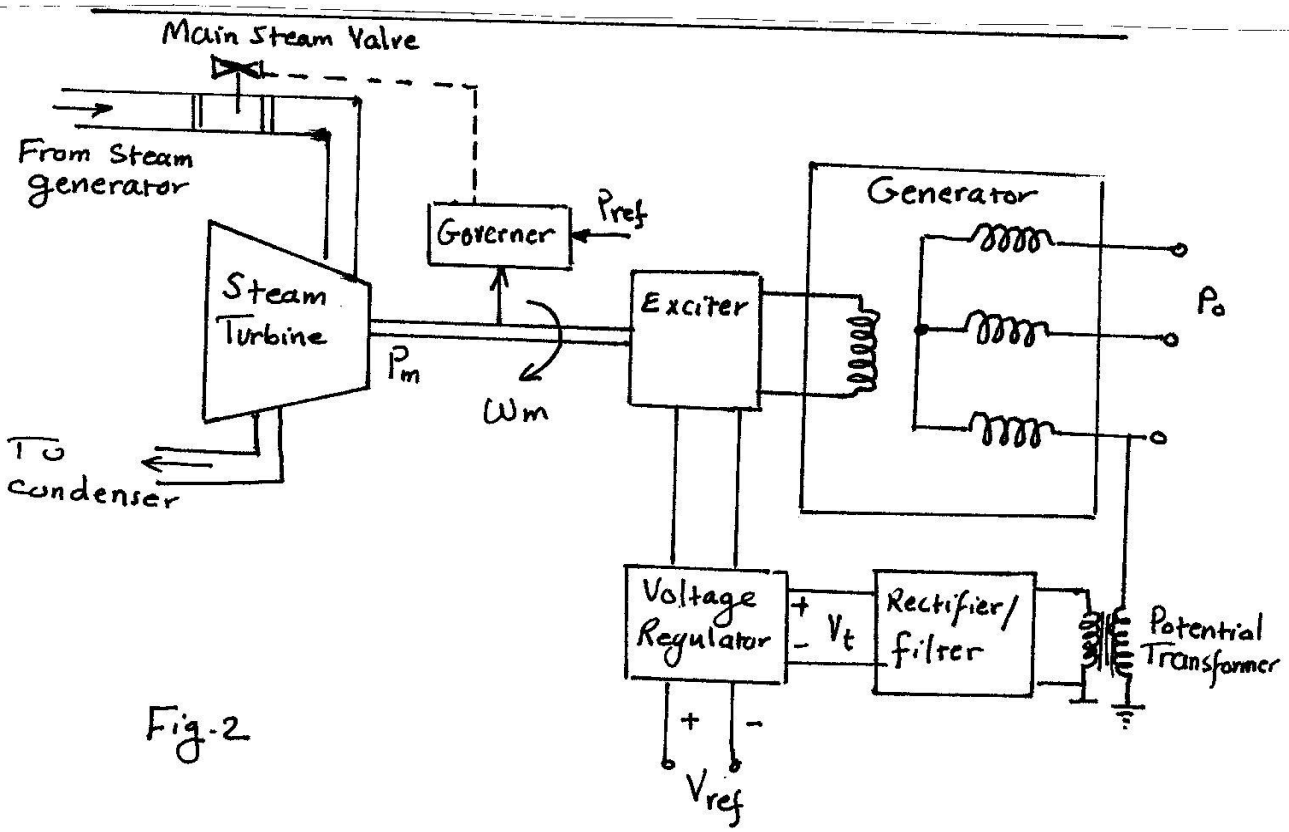


Fig.2

2. Transmission Controls:

These include power and voltage control devices, such as static VAR compensators (STATCOM), synchronous condensers (SC), switched capacitors and reactors, tap changing transformers, phase-shifting transformers and HVDC transmission control.

3. Load Frequency Control:

This includes line or load frequency control. frequency deviation must not be exceeding ±3% or ± 1.5 Hz in the system, hence automatic load frequecy control loops should be used. The frequency error is amplified and fed to speed changer which operates the speed governer to change the speed, hence changing the frequency

$$f = \frac{np}{120} = \frac{p}{120} n = K n$$

Since $K = \frac{p}{120}$ = Constant.

The automatic load frequency control loops (ALFC) and automatic voltage regulator loop (AVR) are shown in Fig.3.

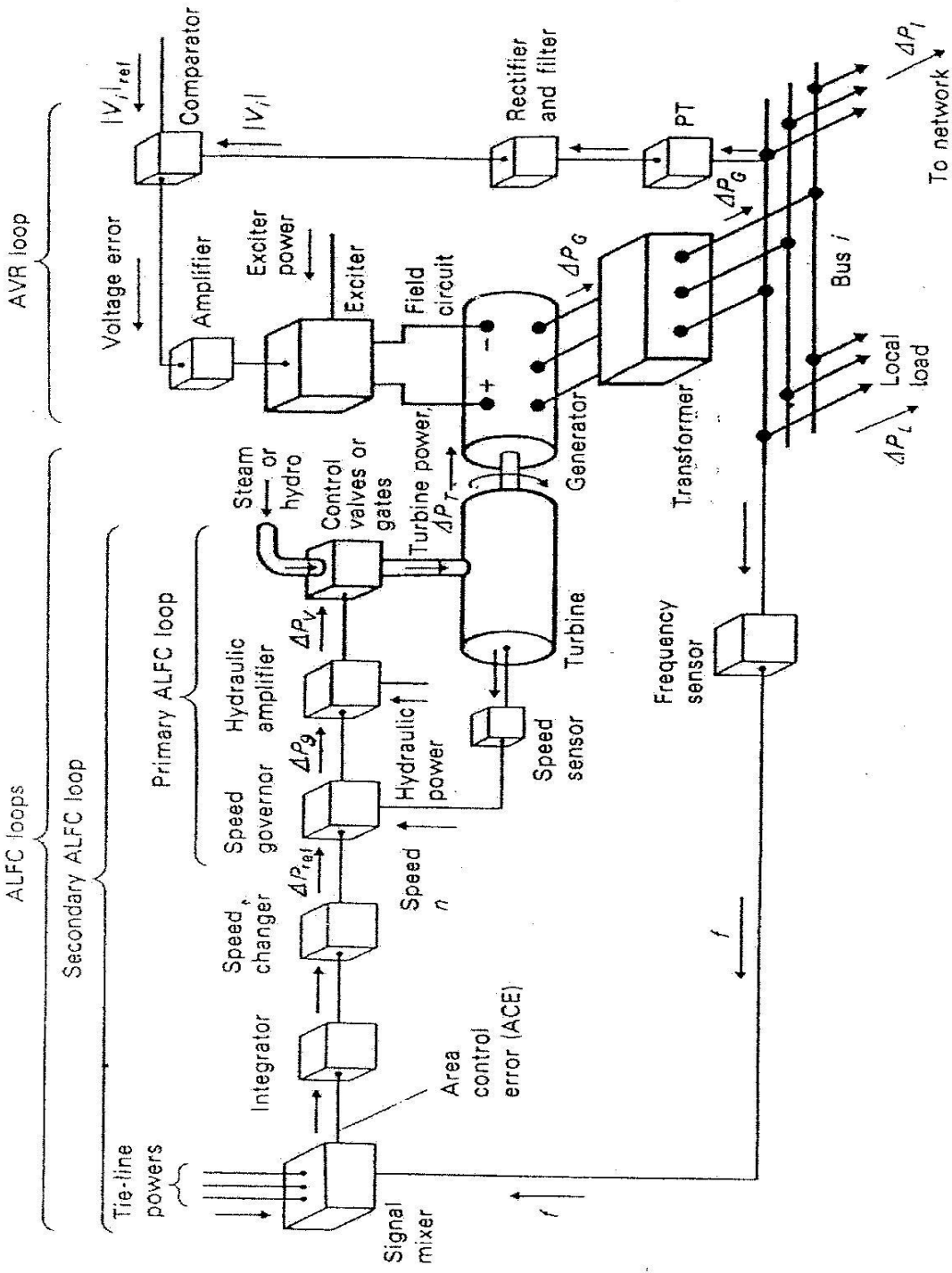


Fig. 3 The automatic load-frequency and voltage regulator control loops of a synchronous generator.

Synchronous Machine Theory and Modeling

- Synchronous generator represents the main source of electric energy in power systems.
- Many large loads are driven by synchronous motors.
- Synchronous condansers are used for providing reactive power compensation and controlling voltage.
  These devices oprate on the same principle and are referred to as synchronous machines.
- The power system stability is to keep the inter-connected synchronous machine oprate in synchronizm.
- The aim of this context is to review the development of the mathamatical model of a synchronous machine and briefly review its steady-state and transient performance characteristics.

Physical Description of Synchronous Machine

Figure.1 shows the schematic of the cross section of a 3-phase synchronous machine with 2-pole field winding. The machine consists of two essential elements: the field and the armature. The field winding carries direct current and produces a magnetic field which induces alternating voltages in the armature winding. In general, there are two types of synchronous machine:

- Salient pole
- Cylindrical rotor.

} see Fig.2.

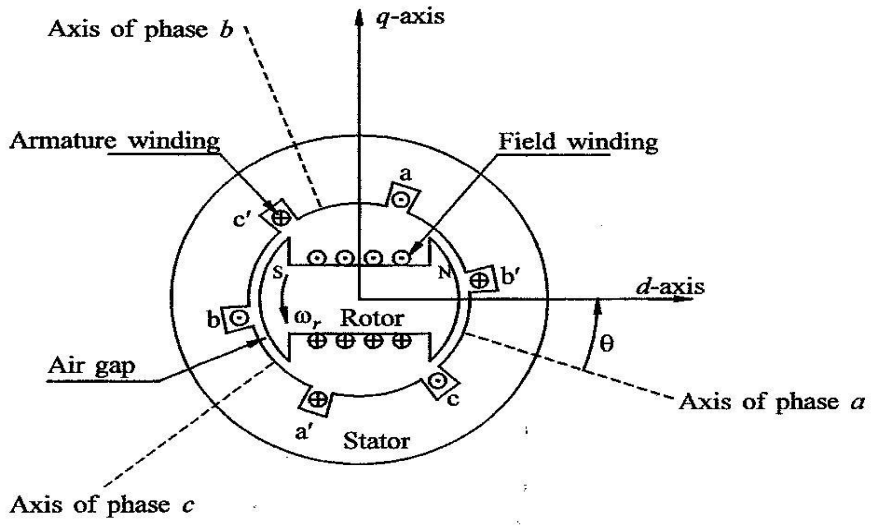


Figure 3.1 Schematic diagram of a three-phase synchronous machine

Direct and Quadrature Axes

In a synchronous machine, two axes are defined as shown in Fig.1 :

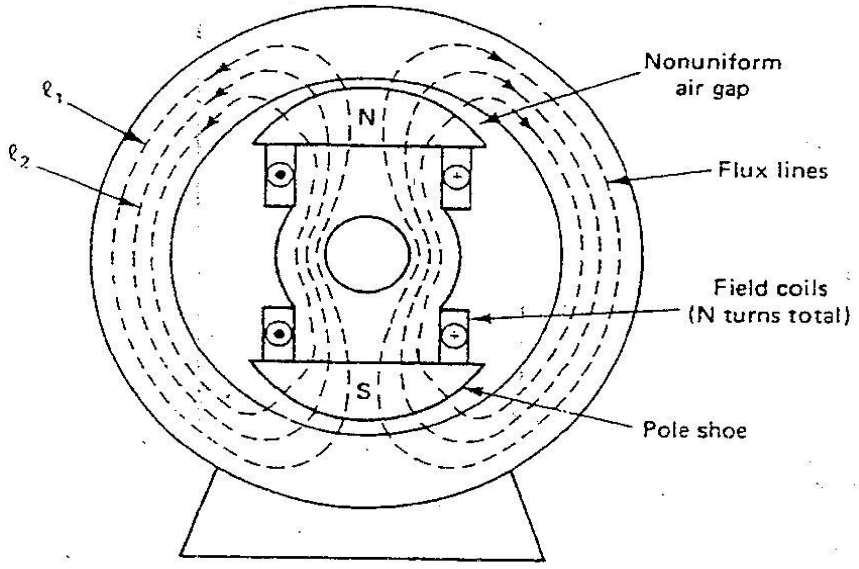
- The direct (d) axis, centred magnetically in the centre of the north pole.
- The quadrature (q) axis, 90 electrical degrees ahead of the d-axis.

The position of the rotor relative to the stator is measured by the angle θ between the d-axis and the magnetic axis of phase (a) winding.

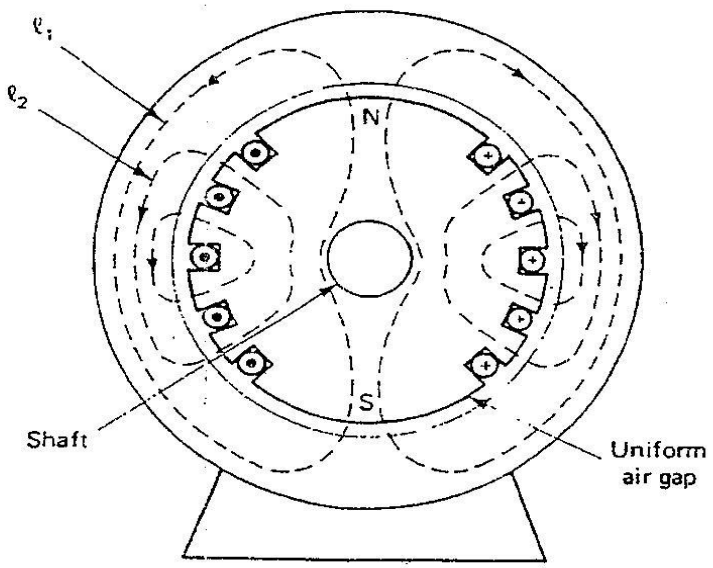
Generator Model in Steady-State (Induced Voltage at No Load).

For the machine shown in Fig.1, when the rotor is excited, it produces an airgap flux of Φ per pole. This flux is revolving at constant angular velocity ω_r. Fig.3 shows the mmf wave of phase a.

Figure 2 **Two-pole synchronous generator: (a) salient pole; (b) cylindrical rotor construction.**



(a)



(b)

© by Prentice Hall Career & Technology
Prentice-Hall, Inc.
A Paramount Communications Company
Englewood Cliffs, New Jersey 07632

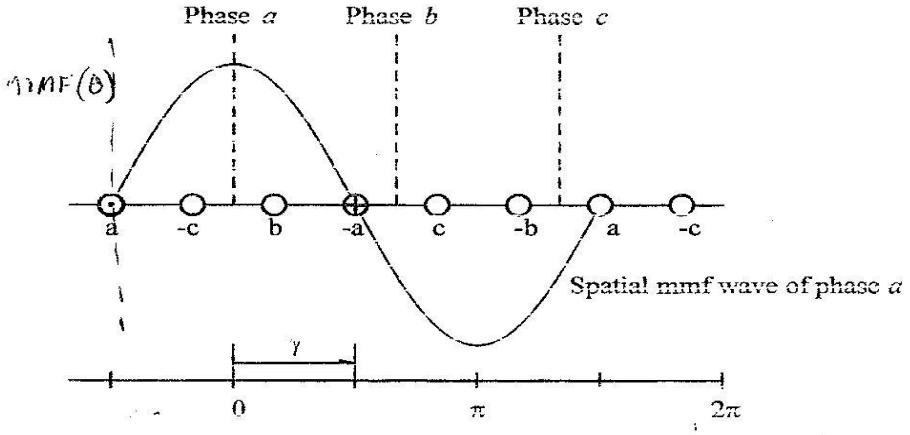


Figure 3 Spatial mmf wave of phase a along the air gap

The flux linkage ψ_a for an N-turn connected coil aa' will be maximum ($N\Phi$) at $\omega t = \theta = 0$, and zero at $\omega t = \pi/2 = \theta$. Hence the flux linkage ψ_a will vary as the cosine of the angle $\omega t = \theta$. Thus the flux linkage with coil a is:

$$\psi_a = N\Phi \cos \omega t$$

The voltage induced in coil aa' is obtained from Faraday's law as

$$e_a = -\frac{d\psi_a}{dt} = \omega N \Phi \sin \omega t = E_{max} \sin \omega t$$
$$= E_{max} \cos\left(\omega t - \frac{\pi}{2}\right).$$

where $E_{max} = \omega N \Phi = 2\pi f N \Phi$

The r.m.s value of the generated e.m.f $E = \frac{E_{max}}{\sqrt{2}}$

$$\therefore E_a = \frac{2\pi}{\sqrt{2}} \cdot f N \Phi = 4.44 f N \Phi.$$

where f is the frequency in Hz which is constant and depends on the rotor speed and on the number of poles, i.e :-

$$f = \frac{P n}{120} \qquad n \Rightarrow \text{synchronous speed.}$$

3/4 For synchronous operation let $\omega = \omega_s$.

Similarly $e_b = E_{max} \sin(\omega t - 120)$ & $e_c = E_{max} \sin(\omega t + 240)$

Synchronous Machine Equivalent Circuit Model:

Steady-state representation of cylindrical rotor machine at load.

- The current I_f in the field produces a flux Φ_f in the air gap.
- When the machine is loaded, the current I_a in the stator winding produces flux Φ_a.

Φ_a — Φ_{al} (leakage flux) links with stator only
Φ_{ar} (armature reaction flux) links with the field winding (major part of Φ_a).

- The resultant air gap flux $\Phi_r = \Phi_f + \Phi_{ar}$.
  Each component Φ_f and Φ_{ar} induces a voltage in the stator winding.

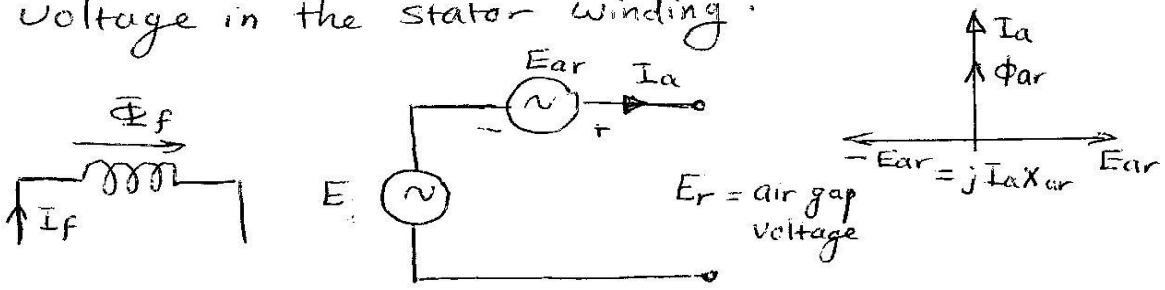
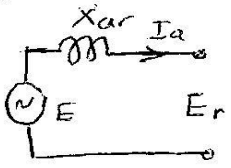


Fig. 1

- Φ_f induces E_r & Φ_{ar} induces E_{ar}.

and $E_r = E_{ar} + E$;

or $E = -E_{ar} + E_r = j I_a X_{ar} + E_r$.



where X_{ar} is the reactance due to armature reaction

- If R_a, the stator winding resistance and X_{al} which is the leakage reactance are included, the per-phase equivalent circuit is shown in Fig. 2

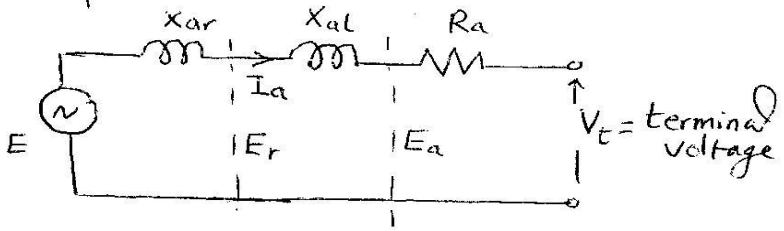


Fig. 2

and the cct of Fig.2 can be simplified as:

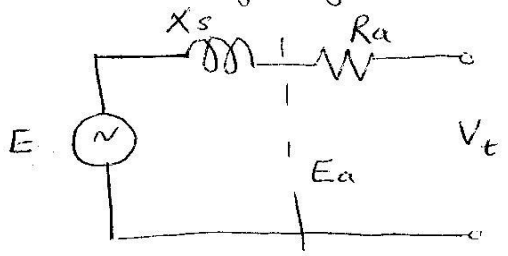


Fig.3

where

$X_s = X_{ar} + X_{al}$ Called synchronous reactance

$Z_s = R_a + jX_s$ Called synchronous impedance.

Table.1 Synchronous Machine Parameters

	Small Machines (tens of kVA)	large Machines (tens of MVA)
R_a	0.05 - 0.02	0.01 - 0.005
X_{al}	0.05 - 0.08	0.1 - 0.15
X_a	0.5 - 0.8	1.0 - 1.5

- R_a is the effective resistance $\simeq$ 1.6 R_{dc} (dc resistance) of the winding.

- Phasor Diagram

From the equivalent cct in Fig.3

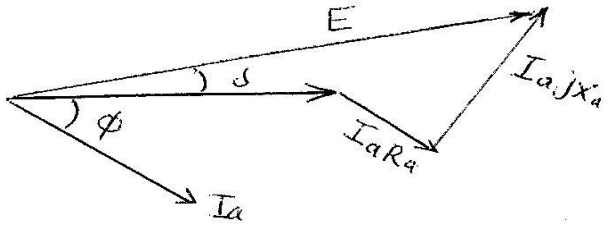
$$E = V_t + I_a R_a + j I_a X_s = |E| \angle \delta .$$

Expressing the quantities in polar form:

$V_t = |V_t| \angle 0^\circ$

$E = |E| \angle \delta$

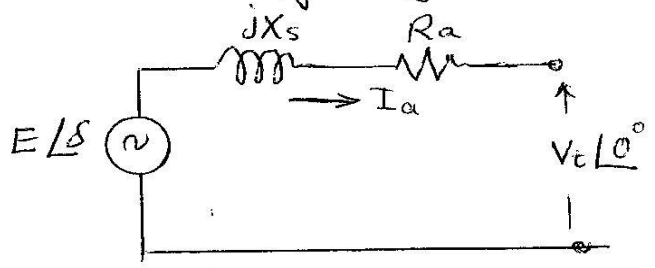
$Z_s = R_a + jX_s = |Z_s| \angle \phi$



δ : is the angle between V_t and E and is called the load angle or rotor angle.

ϕ : is the impedance angle.

The per-phase equivalent circuit of a synch. generator is thus:



- The per-phase complex power S at the terminal is:

$$S = V_t I_a^*$$

In polar form:

$$|E|\angle\delta = |V_t|\angle 0^\circ + I_a|Z_s|\angle\phi$$

$$I_a = \frac{|E|\angle\delta - |V_t|\angle 0^\circ}{|Z_s|\angle\phi}$$

$$= \frac{|E|}{|Z_s|}\angle\delta-\phi - \frac{|V_t|}{|Z_s|}\angle 0^\circ-\phi$$

$$I_a^* = \frac{|E|}{|Z_s|}\angle\phi-\delta - \frac{|V_t|}{|Z_s|}\angle\phi-0^\circ$$

$$S = \frac{|V_t||E|\angle\phi-\delta}{|Z_s|} - \frac{|V_t|^2}{|Z_s|}\angle\phi \quad \text{VA/phase}$$

$$S = \frac{|E||V_t|}{|Z_s|}\{\cos(\phi-\delta) + j\sin(\phi-\delta)\}$$

$$- \frac{|V_t|^2}{|Z_s|}\{\cos\phi + j\sin\phi\}$$

- The real power P and reactive power Q per phase are:

$$P = \mathcal{R}[S] \quad \& \quad Q = I_m[S]$$

$$P = \frac{|E||V_t|}{|Z_s|}\cos(\phi-\delta) - \frac{|V_t|^2}{|Z_s|}\cos\phi \quad \text{Watt/phase}$$

$$Q = \frac{|E||V_t|}{|Z_s|}\sin(\phi-\delta) - \frac{|V_t|^2}{|Z_s|}\sin\phi \quad \text{VAR/phase}$$

If we neglect $R_a \simeq 0$ then $\phi \simeq 90^\circ$ and $Z_s = jX_s$.

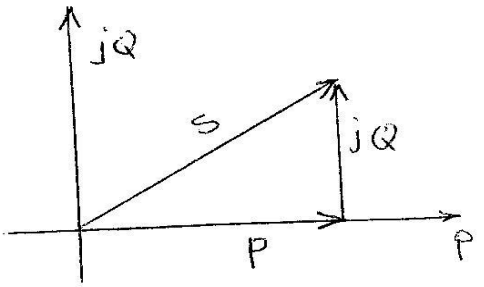
Hence for 3φ machine:

$$P_{3\phi} = \frac{3|E||V_t|}{|X_s|}\sin\delta$$

$$= P_{max}\sin\delta \quad \text{watts}$$

where $P_{max} = \frac{3|E||V_t|}{|X_s|}$

$$Q_{3\phi} = \frac{3|E||V_t|}{|X_s|}\cos\delta - \frac{3|V_t|^2}{|X_s|} \quad \text{VAR}$$



Complex power phasor

- When $\delta = 90^\circ$, $P_G = P_{3\phi}$ = maximum. The machine reaches its stability limit $P_G = P_{max}$. We say that the machine is at pullout.

Torque Characteristics

Neglecting the stator losses, the power developed at the machine terminals is equal to the airgap power. The developed torque of the machine is:

$$T = \frac{P_{3\phi}}{\omega_s} = \frac{3}{\omega_s} \frac{|E||V_t|}{|X_s|} \sin\delta .$$

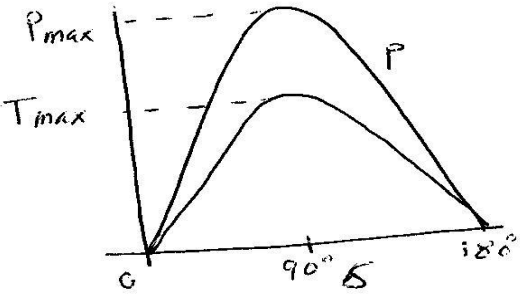
$$= T_{max} \sin\delta \quad N.m.$$

where $T_{max} = \frac{3}{\omega_s} \frac{|E||V_t|}{|X_s|}$

$$\omega_s = \frac{2\pi n_s}{60} .$$

and n_s = synch. speed in rpm.

Both power and torque vary sinusoidally with the angle δ as shown below.



This figure is called the power angle or torque angle curve. The machine can be loaded gradually up to the limit of P_{max} or T_{max} which are known as static stability limits.

- If $\delta > 90^\circ$ the m/c will lose synchronism.
- T_{max} is also called pull-out torque.

3/9

Real and Reactive Power Control of Synchronous Generator

Consider a synchronous generator connected to an infinite bus bar as shown in Fig.1.

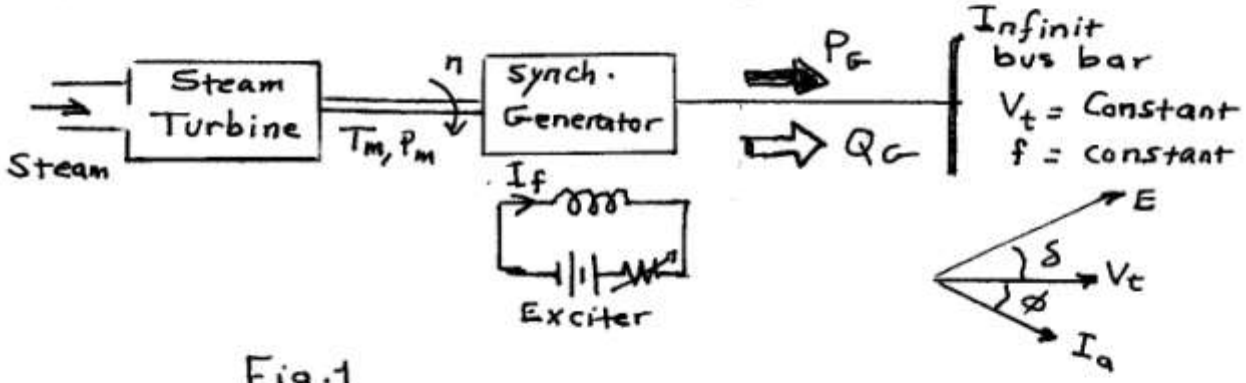


Fig.1

- At normal Conditions : V_t = Constant n = Const. (f = Const.) and I_f = constant.
- To control the real power P_G : This can only be done by variation of the mechanical power P_m input to the generator (Torque Control).
- To control the reactive power Q_G : Q_G is Controlled by variation of I_f (excitation Current). (Excitation Control). Here, we have three cases :
  1. Normal excitation
  2. Over excitation (I_f increases) → generator supplies Q_G to the system.
  3. Under excitation (I_f decreases) → generator draws Q from the system.

Note : This is also applied to Synchronous Motors.

Example 1: A synchronous machine is running over excited with $|E| = 1.5$ p.u. The synchronous reactance $X_s = 1.2$ p.u. and the machine delivers a real power of 0.4 p.u. The terminal voltage is 1.0 p.u, independent of excitation level. If the prime-mover torque is increased by 10%, by how many percent will P_G and Q_G change?.

Solution:

Real and reactive powers when the machine runs over excited:

$$P_G = \frac{|E||V_t|}{X_s}\sin\delta = \underline{\underline{0.4}} \text{ p.u. (given).}$$

$$\frac{1 \times 1.5}{1.2}\sin\delta = 0.4$$

$$\therefore \sin\delta = 0.32 \rightarrow \delta = 18.66^\circ.$$

Hence

$$Q_G = \frac{|V_t||E|}{X_s}\cos\delta - \frac{|V_t|^2}{X_s}$$

$$= \frac{1 \times 1.5}{1.2} \times 0.947 - \frac{1}{1.2} = \underline{\underline{0.35}} \text{ p.u}$$

If the prime-mover torque is increased by 10% P_G will also increased by 10%, hence

$$P_G' = P_G + \frac{10}{100}P_G = 0.4 + \frac{10}{100} \times 0.4 = \underline{\underline{0.44}} \text{ p.u}$$

$$P_G' = \frac{|V_t||E|}{X_s}\sin\delta' = 0.44$$

$$\frac{1 \times 1.5}{1.2}\sin\delta' = 0.44$$

$$\sin\delta' = 0.352 \rightarrow \delta' = 20.6^\circ.$$

$$\therefore Q_G' = \frac{1 \times 1.5}{1.2}\cos 20.6 - \frac{1}{1.2} = \underline{\underline{0.336}}$$

or P_G increased by 4%, while Q_G decreased by 1.6%.

Example 2: A 3-phase, 5 kVA, 208 V, 4-pole, 60 Hz, star connected synchronous machine has negligable stator winding resistance and a synchronous reactance of 8 Ω/phase. The m/c is first operated as a generator and connected to an infinite bus of 208 V.

(a) Determine the excitation voltage and the power angle when the m/c is delivering rated kVA at 0.8 pf lagging. Draw the phasor diagram for this condition.

(b) If the field excitation current is increased by 20% (without changing the prime-mover power), find the stator current, power factor and reactive power supplied by the machine.

(c) With the field current is kept constant as in (a), the prime-mover power is slowly increased. What is the steady-state (or static) stability limit? What are the corresponding values of the stator current, power factor and reactive power Q_G at this maximum power transfer condition.

Solution

The per-phase equivalent circuit of the synch. generator is shown in Fig. (a)

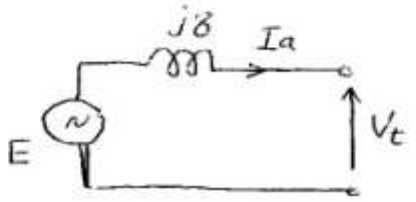


Fig (a)

(a) $V_t = \frac{208}{\sqrt{3}} = 120$ V/phase

$I_a = \frac{S}{\sqrt{3}\, V_{tL}} = \frac{5000}{\sqrt{3} \times 208} = 13.9$ A.

For pf = 0.8 lagging $\quad \phi = \cos^{-1} 0.8 = -36.9^\circ$.

From Fig (a): $E = V_t \angle 0^\circ + j I_a X_s$

$= 120 \angle 0^\circ + j\, 13.4 \angle -36.9^\circ \cdot 8 \angle 90^\circ$

$= 206.9 \angle 25.5^\circ$.

$\therefore \quad \delta = +25.5^\circ$.

The phasor diagram is shown in Fig.(b)

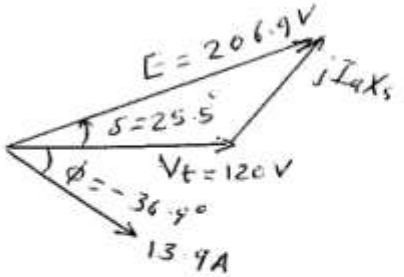
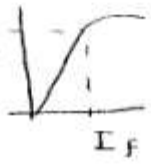


Fig.(b)

(b) The new excitation current = $1.2\,I_f$. hence the new excitation voltage $E' = 1.2E$. (Since $E \propto I_f$).



Hence: $E' = 1.2 \times 206.9 = 248.28\ V$.

Since the power transfer remains same:

$$P_G = P_G'$$

$$\frac{|V_t||E|}{X_s}\sin\delta = \frac{|V_t||E'|}{X_s}\sin\delta'$$

or $$|E|\sin\delta = |E'|\sin\delta'$$

$$\sin\delta' = \frac{|E|}{|E'|}\sin\delta = \frac{\sin 25.5}{1.2}$$

$$\therefore \quad \delta' = 21^\circ.$$

The stator current at this condition will be:

$$I_a = \frac{E' - V_t}{jX_s} = \frac{248.28\angle 21^\circ - 120\angle 0^\circ}{8\angle 90^\circ} = \frac{142.87\angle 38.52^\circ}{8\angle 90^\circ}$$

$$= 17.86\angle -51.5^\circ\ A.$$

$$P.F = \cos 51.5^\circ = 0.62 \text{ lag}$$

Reactive power $Q_G = \frac{3|V_t||E|}{|X_s|}\sin(\phi - \delta) - \frac{3|V_t|^2}{|X_s|}\sin\phi$

$$\therefore \quad Q_G = 3\left(\frac{120 \times 248.28}{8}\sin(90 - 21^\circ) - \frac{120^2}{8}\right)$$

$$= 3(3476.86 - 1800) = 5.03\ KVAr.$$

(C) Maximum power transfer occurs at $\delta = 90^\circ$. Hence:

$$P_{max} = \frac{3|V_t||E|}{|X_s|} = \frac{3 \times 206.9 \times 120}{8} = 9.32 \text{ kW}.$$

$$I_a = \frac{E - V_t}{jX_s} = \frac{206.9 \angle 90^\circ - 120 \angle 0^\circ}{8 \angle 90^\circ}$$

$$= 29.9 \angle 30.1^\circ \text{ A}.$$

$$Pf = \cos 30.1 = 0.865 \text{ lagging}.$$

$$Q_G = \frac{3|V_t||E|}{|X_s|} \cos\delta - \frac{3|V_t|^2}{|X_s|} \text{ VAR}.$$

$$= \frac{3(120)(206)}{8} \cos 90^\circ - \frac{3(120)^2}{8} \text{ VAR}.$$

Real and Reactive Power Control of Synchronous Generator

Consider a synchronous generator connected to an infinite bus bar as shown in Fig.1.

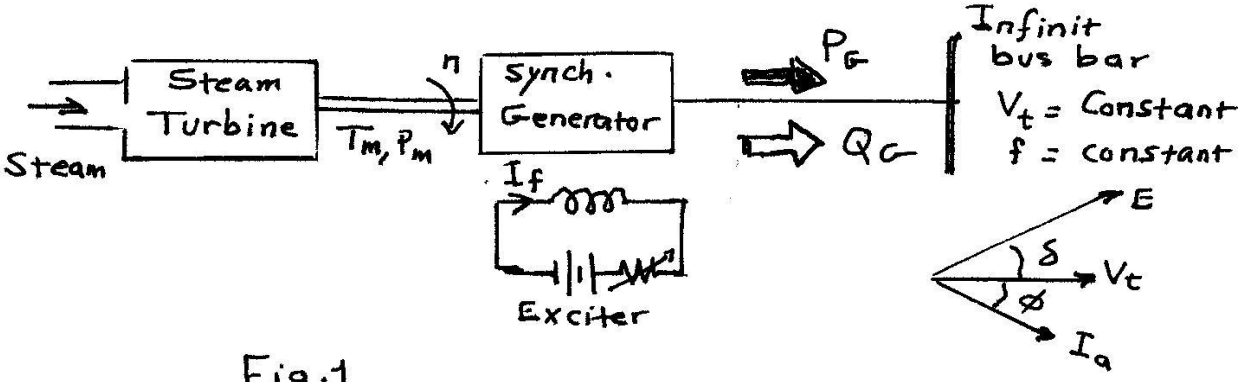


Fig.1

- At normal Conditions : V_t = Constant n = Const. (f = Const.) and I_f = constant.
- To control the real power P_G : This can only be done by variation of the mechanical power P_m input to the generator (Torque Control).
- To control the reactive power Q_G : Q_G is Controlled by variation of I_f (excitation Current). (Excitation Control). Here, we have three cases :
  1. Normal excitation
  2. Over excitation (I_f increases) → generator supplies Q_G to the system.
  3. Under excitation (I_f decreases) → generator draws Q from the system.

Note : This is also applied to Synchronous Motors.

Example 1: A synchronous machine is running over excited with $|E| = 1.5$ p.u. The synchronous reactance $X_s = 1.2$ p.u. and the machine delivers a real power of 0.4 p.u. The terminal voltage is 1.0 p.u, independent of excitation level. If the prime-mover torque is increased by 10%, by how many percent will P_G and Q_G change?.

Solution:

Real and reactive powers when the machine runs over excited:

$$P_G = \frac{|E||V_t|}{X_s}\sin\delta = \underline{0.4} \text{ p.u. (given).}$$

$$\frac{1 \times 1.5}{1.2}\sin\delta = 0.4$$

$$\therefore \sin\delta = 0.32 \rightarrow \delta = 18.66^\circ.$$

Hence $$Q_G = \frac{|V_t||E|}{X_s}\cos\delta - \frac{|V_t|^2}{X_s}$$

$$= \frac{1 \times 1.5}{1.2} \times 0.947 - \frac{1}{1.2} = \underline{0.35} \text{ p.u}$$

If the prime-mover torque is increased by 10% P_G will also increased by 10%, hence

$$P_G' = P_G + \frac{10}{100}P_G = 0.4 + \frac{10}{100} \times 0.4 = \underline{0.44} \text{ p.u}$$

$$P_G' = \frac{|V_t||E|}{X_s}\sin\delta' = 0.44$$

$$\frac{1 \times 1.5}{1.2}\sin\delta' = 0.44$$

$$\sin\delta' = 0.352 \rightarrow \delta' = 20.6^\circ.$$

$$\therefore Q_G' = \frac{1 \times 1.5}{1.2}\cos 20.6 - \frac{1}{1.2} = \underline{0.336}$$

or P_G increased by 4%, while Q_G decreased by 1.6%.

Example 2: A 3-phase, 5 kVA, 208 V, 4-pole, 60 Hz, star connected synchronous machine has negligable stator winding resistance and a synchronous reactance of 8 Ω/phase. The m/c is first operated as a generator and connected to an infinite bus of 208 V.

(a) Determine the excitation voltage and the power angle when the m/c is delivering rated kVA at 0.8 pf lagging. Draw the phasor diagram for this condition.

(b) If the field excitation current is increased by 20% (without changing the prime-mover power), find the stator current, power factor and reactive power supplied by the machine.

(c) With the field current is kept constant as in (a), the prime-mover power is slowly increased. What is the steady-state (or static) stability limit? What are the corresponding values of the stator current, power factor and reactive power Q_G at this maximum power transfer condition.

Solution

The per-phase equivalent circuit of the synch. generator is shown in Fig. (a)

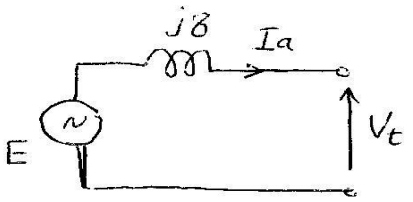


Fig. (a)

(a) $V_t = \frac{208}{\sqrt{3}} = 120$ V/phase

$I_a = \frac{S}{\sqrt{3}\, V_{tL}} = \frac{5000}{\sqrt{3} \times 208} = 13.9$ A.

For pf = 0.8 lagging $\quad \phi = \cos^{-1} 0.8 = -36.9^\circ$.

From Fig (a): $E = V_t \angle 0^\circ + j I_a X_s$

$= 120 \angle 0^\circ + j\, 13.9 \angle -36.9^\circ \cdot 8 \angle 90^\circ$

$= 206.9 \angle 25.5^\circ$.

$\therefore \quad \delta = +25.5^\circ$.

The phasor diagram is shown in Fig.(b)

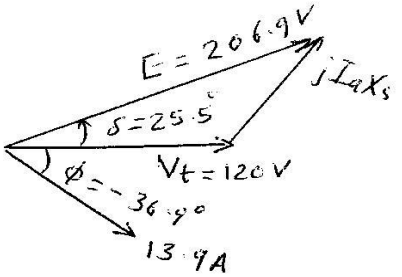
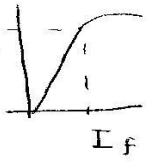


Fig.(b)

(b) The new excitation current = $1.2\,I_f$. hence the new excitation voltage $E' = 1.2E$. (Since $E \propto I_f$).



Hence: $E' = 1.2 \times 206.9 = 248.28\,V$.

Since the power transfer remains same:

$$P_G = P_G'$$

$$\frac{|V_t||E|}{X_s}\sin\delta = \frac{|V_t||E'|}{X_s}\sin\delta'$$

or $$|E|\sin\delta = |E'|\sin\delta'$$

$$\sin\delta' = \frac{|E|}{|E'|}\sin\delta = \frac{\sin 25.5}{1.2}$$

$$\therefore \quad \delta' = 21^\circ.$$

The stator current at this condition will be:

$$I_a = \frac{E' - V_t}{jX_s} = \frac{248.28\angle 21^\circ - 120\angle 0^\circ}{8\angle 90^\circ} = \frac{142.87\angle 38.52^\circ}{8\angle 90^\circ}$$

$$= 17.86\angle -51.5^\circ\ A.$$

$$P.f = \cos 51.5^\circ = 0.62 \text{ lag.}$$

Reactive power $Q_G = \frac{3|V_t||E|}{|X_s|}\sin(\phi_s - \delta) - \frac{3|V_t|^2}{|X_s|}\sin\phi$

$$\therefore \quad Q_G = 3\left(\frac{120 \times 248.28}{8}\sin(90 - 21^\circ) - \frac{120^2}{8}\right)$$

$$= 3(3476.86 - 1800) = 5.03\ KVAr.$$

(C) Maximum power transfer occurs at $\delta = 90^\circ$. Hence:

$$P_{max} = \frac{3|V_t||E|}{|X_s|} = \frac{3 \times 206.9 \times 120}{8} = 9.32 \text{ kW}.$$

$$I_a = \frac{E - V_t}{jX_s} = \frac{206.9 \angle 90^\circ - 120 \angle 0^\circ}{8 \angle 90^\circ}$$

$$= 29.9 \angle 30.1^\circ \text{ A}$$

$$P.f = \cos 30.1 = 0.865 \text{ lagging}.$$

$$Q_G = \frac{3|V_t||E|}{|X_s|} \cos\delta - \frac{3|V_t|^2}{|X_s|} \text{ VAR}.$$

$$= \frac{3(120)(206)}{8} \cos 90^\circ - \frac{3(120)^2}{8} \text{ VAR}.$$

Power System Stability — Dr. Mohammed Tawfeeq

Lecture 5 :

The salient pole Synchronous Machine

- A synchronous machine with salient pole rotor is shown in Fig. 1. This machine has four poles. The stator construction is similar to that of cylindrical rotor machine. The direct and quadrature axes are also shown in Fig. 1.

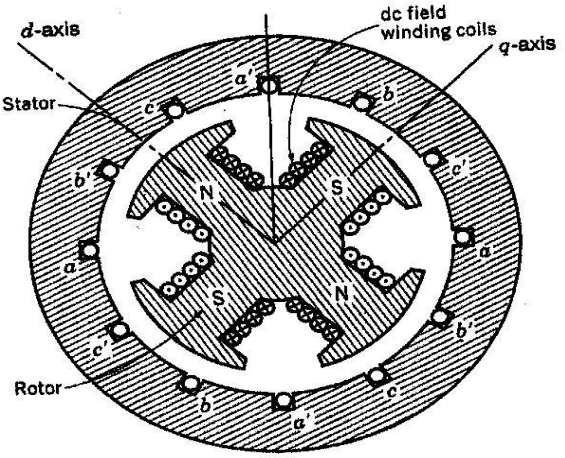
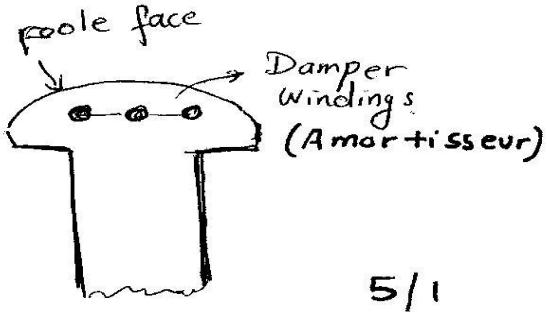


Fig. 1

- Usually the salient-pole machine is provided with damper windings, which consists of short-circuited copper bars embeded through the pole face in the rotor.
- The purpose of the damper winding is to reduce the mechanical oscillations of the rotor about synchronous speed during small perturbation (disturbance).
- The damper winding is also called Amortisseur winding.



5/1

SALIENT-POLE SYNCHRONOUS GENERATORS Analysis :

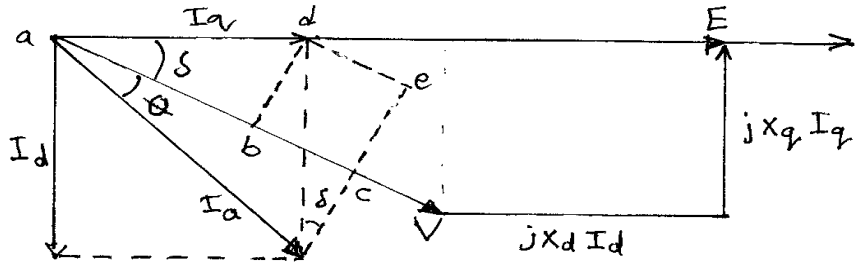
For salient-pole rotor we have:

X_d = Reactance along the direct axis. (high value)

X_q = Reactance along the quadrature axis. (Low value).

These reactances produce voltage drop in the armature and can be taken into account by resolving the armature current I_a into two components I_q which is in-phase with E and I_d in time quadrature with E.

The phasor diagram is given in the following Figure with Ra neglected.



$$|E| = |V| \cos\delta + X_d I_d \quad \ldots\ldots (1)$$

The 3-phase real power at the generator terminal is:

$$P = 3|V||I_a|\cos\phi \quad \ldots\ldots (2)$$

but $|I_a|\cos\phi = ab + bc = ab + de = I_q \cos\delta + I_d \sin\delta \quad \ldots (3)$

(since $bc = dc$)

Substituting (3) into (2) we have

$$P = 3|V|(I_q \cos\delta + I_d \sin\delta) \quad \ldots\ldots (4)$$

but $|V| \sin\delta = X_q I_q \quad \ldots\ldots (5)$

or $$I_q = \frac{|V|\sin\delta}{X_q} \quad \ldots\ldots (6)$$

Also from eq.(1):

$$I_d = \frac{|E| - |V|\cos\delta}{X_d} \quad \text{- - - - - (7)}$$

Subst for I_d and I_q into eq.4:

$$\therefore P_{3\phi} = \frac{3|E||V|}{X_d}\sin\delta + 3|V|^2 \frac{X_d - X_q}{2X_d X_q}\sin 2\delta \quad \text{--(8)}$$

(Reluctance power)

- It is clear that from eq.(8), the maximum power occures at $\delta < \frac{\pi}{2}$. Hence the salient-pole machine is more stable than the cylindrical rotor machine

- At steady-state we can not operate the cylindrical rotor generator at the stability limit or near it where $\delta \simeq \frac{\pi}{2}$.

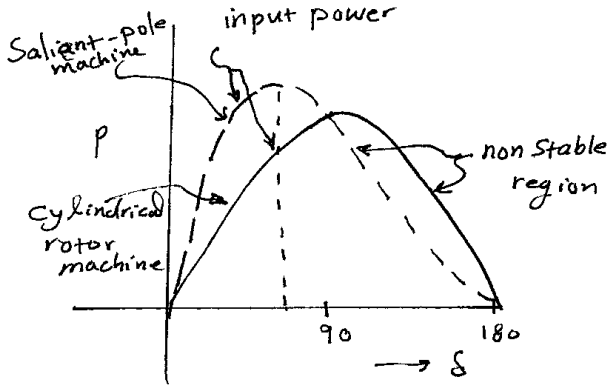


Fig. Variation of power for synchronous generator at steady-state both for cylindrical rotor and salient pole type.

Example : A 6.6 kV, 5 MVA, 6-pole, 50 Hz star-connected synchronous generator with salient-pole rotor. The generator has $X_d = 8.7\,\Omega$ per phase and $X_q = 4.35\,\Omega$ per phase. Resistance may be neglected. If the excitation is so adjusted that $E = 11$ kV (line), and the load angle $\delta = 30^\circ$, determine :

(a) The armature output current, power, reluctance power and power factor.

$$I_a = \frac{S}{\sqrt{3}V} = \frac{5000 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3} = 437.4\ A = 1.0\ pu.$$

$$I_a = \frac{V \sin\delta}{X_q} = \frac{\frac{6600}{\sqrt{3}} \sin 30^\circ}{4.35} = 438\ A.$$

$$I_d = \frac{E - V\cos\delta}{X_d} = \frac{11000 - 6600\cos 30^\circ}{\sqrt{3} \times 8.7} \quad \text{(per phase)}$$

$$= 350.7\ A.$$

$$I_a = \sqrt{I_a^2 + I_q^2} = \sqrt{(437.4)^2 + (350.7)^2} = 561.1\ A.$$

$$\tan\psi = \frac{I_d}{I_q} = \frac{350}{438}, \text{ so } \psi = 38.7^\circ$$

$$\psi = \delta + \phi \qquad \therefore \ \phi = \psi - \delta = 38.7^\circ - 30 = 8.7^\circ$$

$$\therefore \cos\phi = p.f = \cos 8.7 = 0.988 \text{ lagging.}$$

$$\text{Power} = \sqrt{3} V I_a \cos\phi = \sqrt{3} \times 6600 \times 561.1 \times 0.988$$

$$= 6.34\ MW.$$

or

$$P_{3\phi} = \frac{3|E||V|}{X_d} \sin\delta + 3|V|^2 \frac{X_d - X_q}{2X_d X_q} \sin 2\delta$$

$$= \frac{3 \frac{11000}{\sqrt{3}} \times \frac{6600}{\sqrt{3}}}{8.7} \sin 30^\circ + 3\left(\frac{6600}{\sqrt{3}}\right)^2 \frac{8.7 - 4.35}{2 \times 8.7 \times 4.35} \sin 60^\circ$$

$$= 4.1720 + 2.168 = 6.340\ MW$$

Reluctance power $= 2.168$ MW.

(b) The load angle at maximum torque.

$$T = \frac{P}{\omega_s} = \frac{1}{\omega_s} \times \frac{|V|}{X_d}\left[|E|\sin\delta + \frac{|V|}{2}\left(\frac{X_d}{X_q} - 1\right)\sin 2\delta\right]$$

For maximum torque $\frac{dT}{d\delta} = 0$.

$$0 = \frac{1}{\omega_s} \times \frac{|V|}{X_d}\left[|E|\cos\delta + \frac{2|V|}{2}\left(\frac{X_d}{X_q} - 1\right)\cos 2\delta\right]$$

$$\therefore \quad E\cos\delta = -|V|\left(\frac{X_d}{X_q} - 1\right)\cos 2\delta = -|V|\left(\frac{X_d}{X_q} - 1\right)(2\cos^2\delta - 1)$$

or $$2\cos^2\delta + \frac{|E|}{|V|}\left(\frac{X_q}{X_d - X_q}\right)\cos\delta - 1 = 0.$$

$$\therefore \quad \cos\delta = -\frac{|E|}{4|V|}\left(\frac{X_q}{X_d - X_q}\right) \pm \sqrt{\left(\frac{|E|}{4V}\right)^2\left(\frac{X_d}{X_d - X_q}\right) + \frac{1}{2}}$$

or $$\cos\delta = \frac{-11}{4 \times 6.6}\left(\frac{4.35}{4.35}\right) \pm \sqrt{\frac{121}{26.4^2} + 0.5}$$

$$= -0.4135 \quad \pm 0.8207$$

$$\therefore \quad \cos\delta = 0.4072 \quad \therefore \delta = 66^\circ$$

Synchronous Machine Transient Analysis

In order to understand the synchronous machine transient, let us consider the behavior of the simple RL circuit, shown in Fig.1, switched suddenly to a.c. voltage $v(t) = V_m \sin(\omega t + \alpha)$.

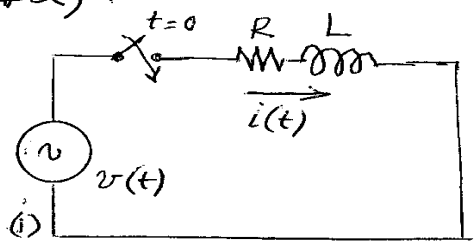


Fig.1

By KVL:

$$R i(t) + L \frac{d i(t)}{dt} = v(t) = V_m \sin(\omega t + \alpha) \quad \text{-- (1)}$$

The solution of this equation contains two solutions:

- The steady-state solution $\Rightarrow i_{ss}$
- The transient solution $\Rightarrow i_{tr}$

or:

$$i(t) = i_{ss} + i_{tr} = \frac{V_m}{Z} \sin(\omega t + \alpha - \phi) + A e^{-\frac{R}{L} t} \quad \text{...(2)}$$

The current waveform is shown in Fig.2

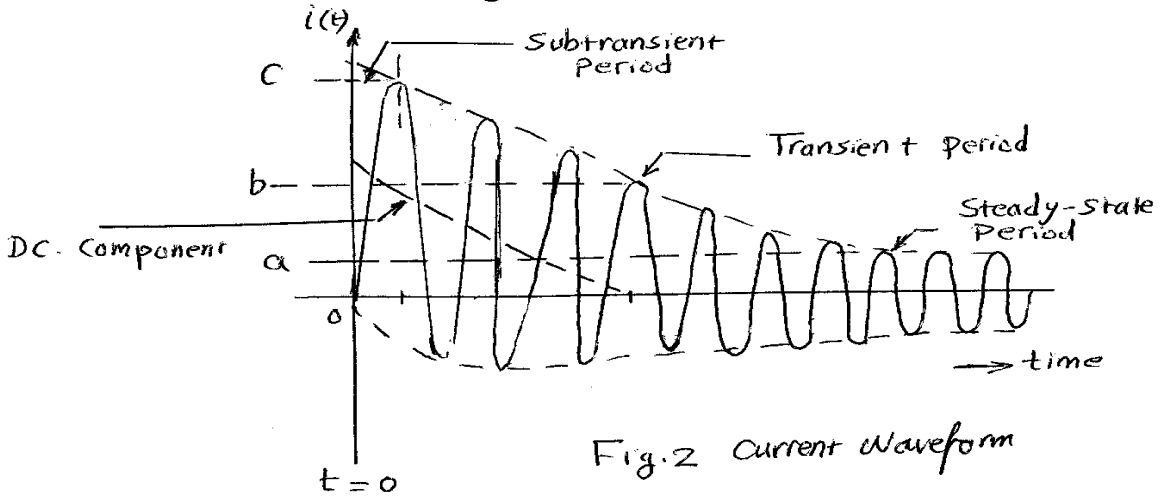


Fig.2 Current Waveform

Where : $Z = \sqrt{R^2 + (\omega L)^2}$

$\phi = \tan^{-1} \frac{\omega L}{R}$

A = constant which can be evaluated from initial conditions as follows:

at $t = 0$ $\quad i(0) = 0$.

Substitute thes values in eq.(2):

$$0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A$$

$$\therefore A = \frac{-V_m}{Z} \sin(\alpha - \phi)$$

Hence the final form of the current equation is:

$$i(t) = \frac{V_m}{Z}\left[\sin(\omega t + \alpha - \phi) - \sin(\alpha - \phi)\, e^{-\frac{t}{\tau}}\right] \quad \text{---(3)}$$

where τ = time constant $= \frac{L}{R}$.

The first term of eq.(3) is the steady-state sinusoidal component and the second term is the DC transient component

Short circuit on the terminal of a Synchronous Generator

When a sudden short-circuit occurs on the terminal of a synchronous generator, same current waveform as in Fig.3 produced. Hence during short-circuit period we can define three types of current components:

- I_a'' = Subtransient Component $= \frac{OC}{\sqrt{2}}$
- I_a' = Transient Component $= \frac{Ob}{\sqrt{2}}$

- I_a = Steady-State Component $= \frac{0a}{\sqrt{2}}$.

Also we shall have three types of reactances associated with each component of the current as follows:

- X_d'' = Direct-axis Subtransient reactance $= \frac{V_{rms}}{I_a''} = \frac{V_m/\sqrt{2}}{I_a''}$
- X_d' = Direct-axis transient reactance $= \frac{V_{rms}}{I_a'} = \frac{V_m}{\sqrt{2} I_a'}$
- X_d = Direct-axis synchronous reactance $= X_s = \frac{V_m}{\sqrt{2} I_a}$.

The equivalent circuit for each period is shown in Fig. 4

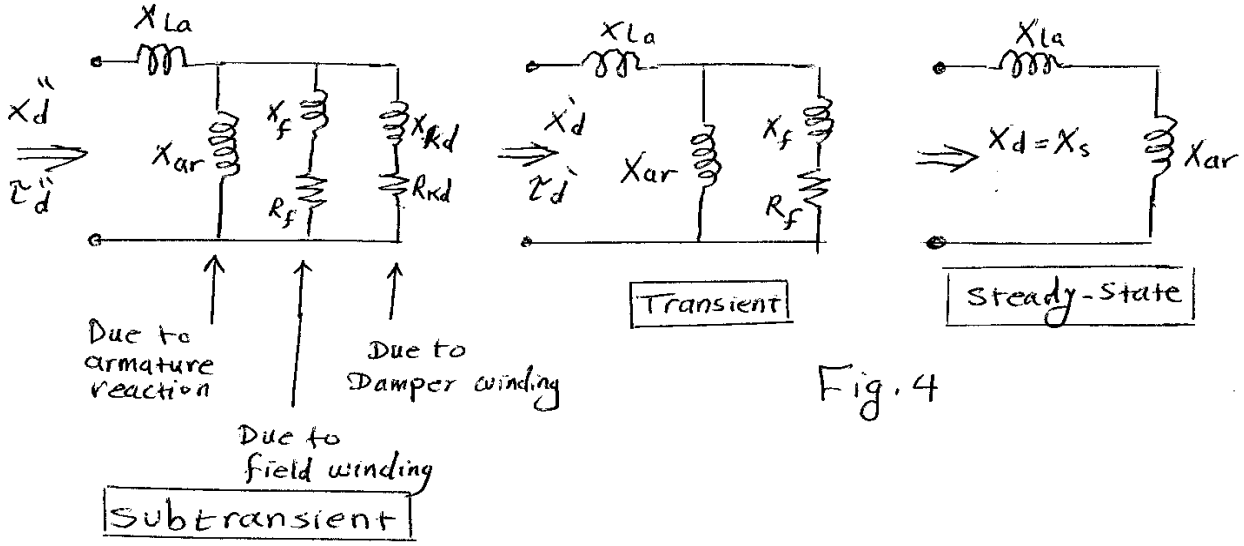


Fig. 4

Also we have:

- T_d'' = Direct-axis short-circuit time constant
  $\simeq$ 35 mSec.
- T_d' = Direct-axis short-circuit time constant
  $\simeq$ (1-2) sec.
- T_d = Direct-axis short-circuit time constant
  $\simeq \infty$ (since we don't have R in the circuit).

In similar manner to RL circuit, the short circuit current of unloaded synchronous generator can be expressed as:

$$i_{sc}(t) = i_{ac}(t) + I_{dc} .$$

where

- $$i_{ac}(t) = V_m \left[\left(\frac{1}{x_d''} - \frac{1}{x_d'} \right) e^{-\frac{t}{\tau_d''}} + \left(\frac{1}{x_d'} - \frac{1}{x_d} \right) e^{-\frac{t}{\tau_d'}} + \frac{1}{x_d} \right] \sin(\omega t + \delta)$$

This is called the AC component.

- $$I_{dc} = \frac{V_m}{x_d''} \sin\delta \; e^{-\frac{t}{\tau_a}} .$$

This is called the transient or DC component

where τ_d = armature short circuit time constant

$$\simeq \frac{x_d'' + x_q''}{2R_a} \simeq 100 \text{ ms}.$$

Note: The worest possible transient condition is when $\delta = 90^\circ$. and the maximum possible initial magnitude of the d.c. Component is

$$I_{dc\,(max)} = \frac{V_m}{x_d''}$$

The total equation of the S.C. current is

$$i_{sc}(t) = V_m \left[\left(\frac{1}{x_d''} - \frac{1}{x_d'} \right) e^{-\frac{t}{\tau_d''}} + \left(\frac{1}{x_d'} - \frac{1}{x_d} \right) e^{-\frac{t}{\tau_d'}} + \frac{1}{x_d} \right] \sin(\omega t + \delta) + \frac{V_m}{x_d''} \sin\delta \; e^{-\frac{t}{\tau_a}} \quad \text{---- (4)}$$

From eq.(4) we can deduce that, for a synch. generator:

$$I_d'' = \frac{V_m/\sqrt{2}}{X_d''}$$ rms value of the subtransient S.C. current component.

$$I_d' = \frac{V_m/\sqrt{2}}{X_d'}$$ rms value of transient S.C. current component

$$I_d = \frac{V_m/\sqrt{2}}{X_d}$$ rms value of the steady-state S.C. current component

Therefore, at the begining of short circuit, the maximum rms current (ac+dc) is

$$I_{asy} = \sqrt{(I_d'')^2 + (I_{dc})^2}$$

This is called the asymmetrical short circuit current, or we can write it as:

$$I_{asy} = \sqrt{\left(\frac{V_m}{\sqrt{2}\,X_d''}\right)^2 + \left(\frac{V_m}{X_d''}\right)^2} = \frac{V_m}{X_d''}\sqrt{\frac{1}{2}+1}$$

$$= \sqrt{\frac{3}{2}}\left(\frac{V_m}{X_d''}\right) = \sqrt{3}\, I_d''$$

Note: In practice, the momentary duty of the circuit breaker is given by its asymmetrical short-circuit current.

Example: A 100 MVA, 13.8 kV, 60 Hz, Y-connected, 3-phase synchronous generator is connected to 13.8 kV/220 kV, 100 MVA, Δ-Y connected transformer. The reactances in per unit to the machine's own base are:

$X''_d = 0.12$ p.u, $X'_d = 0.25$ p.u., $X_d = 1.0$ p.u.

and its time constants are:

$\tau''_d = 0.4$ sec., $\tau'_d = 1.1$ sec., $\tau_a = 0.25$ sec.

The transformer reactance is 0.20 p.u. on the same base. The generator is operating at the rated voltage and no-load when a 3-phase fault occurs at the secondary terminals of the transformer as shown in Fig.1.

(a) Find the subtransient, transient and the steady-state short circuit currents in per unit and actual amperes on both sides of the transformer.

Solution: The base current on the generator side:

$$S = \sqrt{3}\, V_L I_L$$

$$I_{b_1} = \frac{S_b}{\sqrt{3}\, V_{b_1}} = \frac{100 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3}$$

$$= 4184 \text{ A}$$

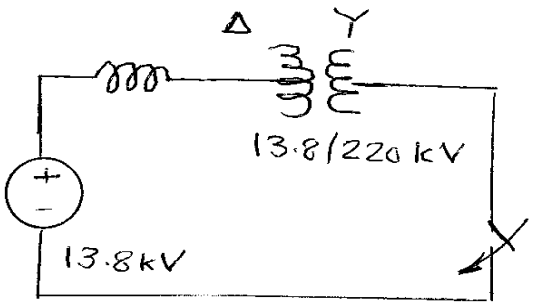


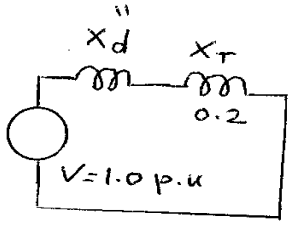
Fig.1

The base current on the secondary side of the transformer:

$$I_{b_2} = \frac{V_p}{V_s} I_{b_1} = \frac{13.8}{220} \times 4184 = 262.4 \text{ A}$$

The subtransient, transient and the steady-state S.C. currents are calculated as follows:

$$I_d'' = \frac{1.0}{0.12+0.2} = 3.125 \text{ p.u.}$$



On generator side :

$I_d'' = 3.125 \times 4184 = 13075 \text{ A}$

On the 220 kV side of the transformer:

$I_d'' = 262.4 \times 3.125 = 820 \text{ A}$

Similarly :

$I_d' = \frac{1.0}{0.25+0.2} = 2.22 \text{ p.u.} = 9288.5 \text{ A}$ on the generator side.

$= 582.5 \text{A}$ on the 220kV side.

$I_d = \frac{1.0}{1+0.2} = 0.833 \text{ p.u.} = 3486 \text{A}$ on the generator side.

$= 218.6 \text{ A}$ on the 220kV side.

(b) Find the assymetrical r.m.s current (ac+dc) at the begining of the fault.

$I_{asy} = \sqrt{3}\, I_d'' = \sqrt{3}(3.125) = 5.4 \text{ p.u.}$

$= 22.64 \text{ kA}$ on the generator side.

(c) Find the instantaneous short-circuit current in terms of I_d'', I_d', I_d if $\delta = \pi/2$.

$$i_{sc}(t) = V_m\left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)e^{-\frac{t}{t_d''}} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)e^{-\frac{t}{t_d'}}\right]\sin(\omega t+\delta) + \frac{V_m}{X_d''}\sin\delta \cdot e^{-\frac{t}{\tau_d}}$$

$$= \sqrt{2}\left[\left(\frac{V}{X_d''} - \frac{V}{X_d'}\right)e^{-\frac{t}{t_d''}} + \left(\frac{V}{X_d'} - \frac{V}{X_d}\right)e^{-\frac{t}{t_d'}} + \frac{V}{X_d}\right]\sin\left(\omega t+\frac{\pi}{2}\right) + \frac{\sqrt{2}V}{X_d''}\sin\frac{\pi}{2}\, e^{-\frac{t}{\tau_d}}$$

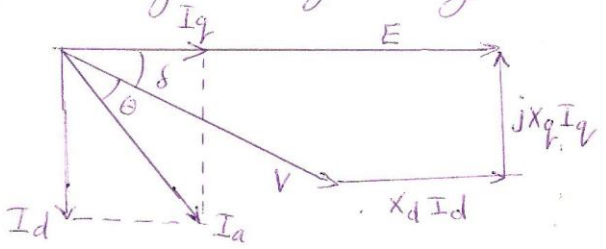
$$= \sqrt{2}\left[(I_d'' - I_d')\, e^{-\frac{t}{0.4}} + (I_d' - I_d)e^{-\frac{t}{0.1}} + I_d\right]\sin\left(\omega t+\frac{\pi}{2}\right) + \sqrt{2}\, I_d''\, e^{-t/0.25}$$

$$i_{sc}(t) = \left[1.28\, e^{-2.5t} + 1.9\, e^{-0.91t} + 1.18\right]\sin(377t+90^\circ) + 4.42\, e^{-4t} \text{ p.u.}$$

Power System Stability	Lecture : 7	Dr. Mohammed Tawfeeq
Load Angle Calculations for Salient pole Machine		

1. During Steady State :- Machine Modeling :

Under steady-state condition, the salient-pole machine phasor diagram is given by:



The power-angle equation was given by:

$$P_e = \frac{|E||V|}{X_d} \sin\delta + |V|^2 \frac{X_d - X_q}{2X_d X_q} \sin 2\delta \quad \text{p.u.} \quad (1)$$

and $|E| = |V|\cos\delta + X_d I_d$ - - - - -

$$|E| = |V|\cos\delta + X_d |I_a| \sin(\delta+\theta) \quad (2)$$

where E is the no-load generated emf in p.u.
V is the generator terminal voltage in p.u.
X_d and X_q are the direct and quadrature axis reactances of the synchronous machine.

Computing δ :

$$|V|\sin\delta = X_q I_q$$
$$= X_q |I_a| \cos(\delta+\theta)$$
$$= X_q |I_a| [\cos\delta\cos\theta - \sin\delta\sin\theta]$$

From this relation we can find δ :

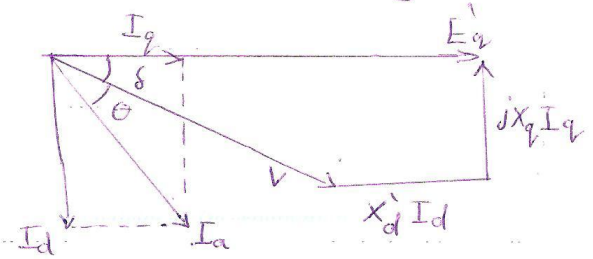
$$\delta = \tan^{-1} \frac{X_q |I_a| \cos\theta}{|V| + X_q |I_a| \sin\theta} \quad (3)$$

Substituting (3) into (2) we can find E.

Salient-pole Synch. Machine Modeling

2. During transient period

The direct axis transient reactance becomes X_d'. Since the field is on the direct axis X_q' the quadrature axis transient reactance remains the same as X_q. Hence the phaser diagram becomes:



The transient power-angle equation expressed in p.u. becomes:

$$P_e = \frac{|E_q'||V|}{X_d'} \sin\delta + |V|^2 \frac{X_d' - X_q}{2X_d' X_q} \sin 2\delta \quad \text{---(4)}$$

$$|E_q'| = |V|\cos\delta + X_d' I_d \quad \text{---(5)}$$

$$|E_q'| = |V|\cos\delta + X_d' |I_a| \sin(\delta+\theta) \quad \text{---(6)}$$

From eq(2), we find:

$$|I_a| \sin(\delta+\theta) = \frac{|E| - |V|\cos\delta}{X_d}$$

and substituting it in the above equation to get

$$|E_q'| = \frac{X_d'|E| + (X_d - X_d')\cos\delta}{X_d} \quad \text{---(7)}$$

The prefault excitation voltage and power angles are computed from (2) and (3).

Example : A synchronous machine has the following parameters :

$X_d = 1 \quad X_q = 0.6 \quad X'_d = 0.3$ p.u. , $R_a = 0$.

The machine is connected directly to an infinite bus of $V = 1.0$ p.u. The generator is delivering a real power of 0.5 p.u. at 0.8 p.f lagging. Determine the voltage behind transient reactance and the transient power-angle equation for the following cases:

(a) Neglecting the saliency effect.

(b) Including the effect of saliency.

Solution

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

$$S = \frac{0.5}{0.8} \angle 36.87^\circ = 0.625 \angle 36.87^\circ \text{ p.u.}$$

The prefault steady-state current is

$$I_a = \frac{S^*}{V^*} = 0.625 \angle -36.87^\circ \text{ p.u.}$$

(a) With saliency neglected, the voltage behind transient reactance is

$$E' = V + jX'_d I_a = 1.0 + (j0.3)(0.625 \angle -36.87^\circ)$$

$$= 1.1226 \angle 7.679^\circ \text{ p.u.}$$

The transient power-angle curve is given by

$$P_e = \frac{|E'||V|}{X'_d} \sin\delta = \frac{(1.1226)(1)}{0.3} \sin\delta.$$

$$P_e = 3.7419 \sin\delta. \quad \text{------(1)}$$

(b) When the saliency effect is considered, the initial steady-state power angle is

$$\delta = \tan^{-1}\frac{X_q|I_a|\cos\theta}{|V|+X_q|I_a|\sin\theta} = \tan^{-1}\frac{(0.6)(0.625)(0.8)}{1+(0.6)(0.625)(0.6)}$$

$$= 13.76^\circ .$$

The steady-state excitation voltage E, is

$$|E| = |V|\cos\delta + X_d|I_a|\sin(\delta+\theta)$$

$$= 1.0\cos(13.76^\circ) + (1.0)(0.625)\sin(13.76^\circ + 36.87^\circ) = 1.45 \text{ p.u}$$

The transient voltage E_q' is

$$|E_q'| = \frac{X_d'|E| + (X_d - X_d')|V|\cos\delta}{X_d} .$$

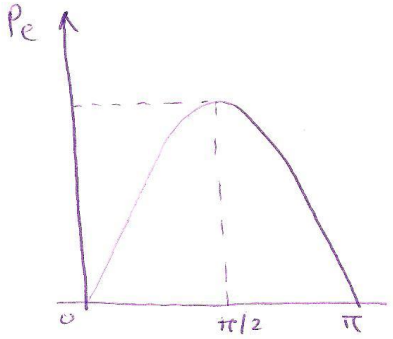
$$= \frac{(0.3)(1.9545) + (1.0-0.3)(1.0)(\cos 13.76)}{1.0} = 1.1162 \text{ pu}$$

and from the transient power-angle is

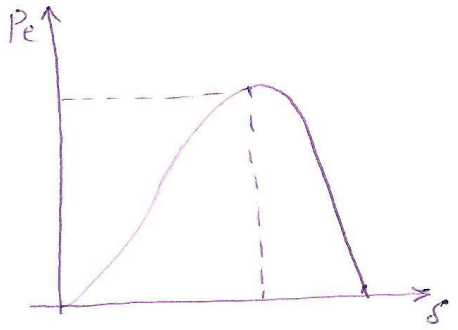
$$P_e = \frac{(1.1162)(1)}{0.3}\sin\delta + \frac{(1.0)^2(0.3-0.6)}{2(0.3)(0.6)}\sin 2\delta .$$

$$P_e = 3.72\sin\delta - 0.833\sin 2\delta . \quad ----(2)$$

plots of eq.(1) and (2) are shown below:



eq.(1)



Eq.(2)

Lecture: 8

Stability of Power System	Dr. Mohammed Tawfeeq
Review of Synchronous Machines	

Power-Angle characteristic of Synch. Machine

Fig. 1 below shows the steady-state real power variation with power angle for both synch. generator and synch. motor action for constant values of E, V_t and X_s (or X_d).

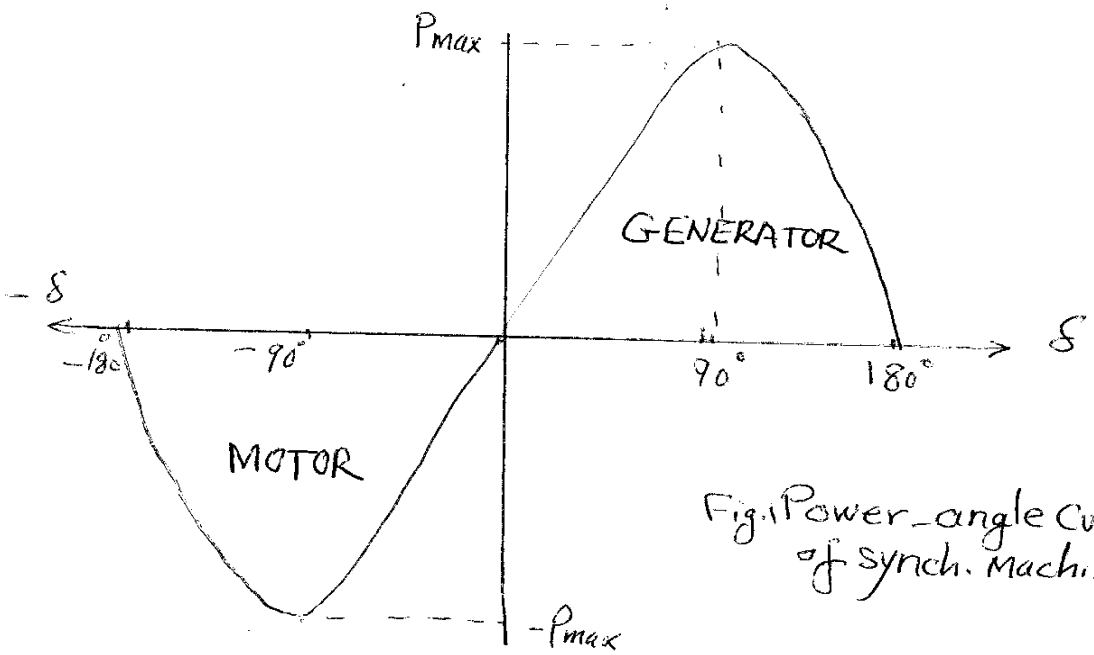


Fig. 1 Power-angle Curve of Synch. Machine

This curve is known as power-angle curve. The condition of positive values of δ, i.e E leading V_t applied to generator action and the condition of negative value of δ, i.e E lagging V_t applied to motor action.

Synchronizing Power and Synchronizing Power Coefficient "P_s"

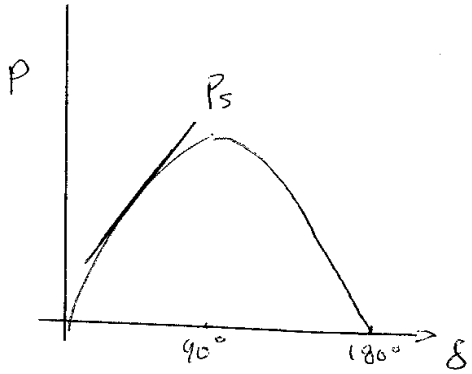
Assume that a synch generator is working under steady-state conditions and the power angle δ increased by small amount $\Delta\delta$. The increase in the synchronous power output is:

$$\Delta P = \frac{dP}{d\delta}(\Delta\delta) = P_s(\Delta\delta)$$

where $$P_s = \frac{dP}{d\delta} = \frac{3|E||V|}{X_s}\frac{d}{d\delta}(\sin\delta)$$

or $$\boxed{P_s = \frac{3|E||V|}{X_s}\cos\delta}$$

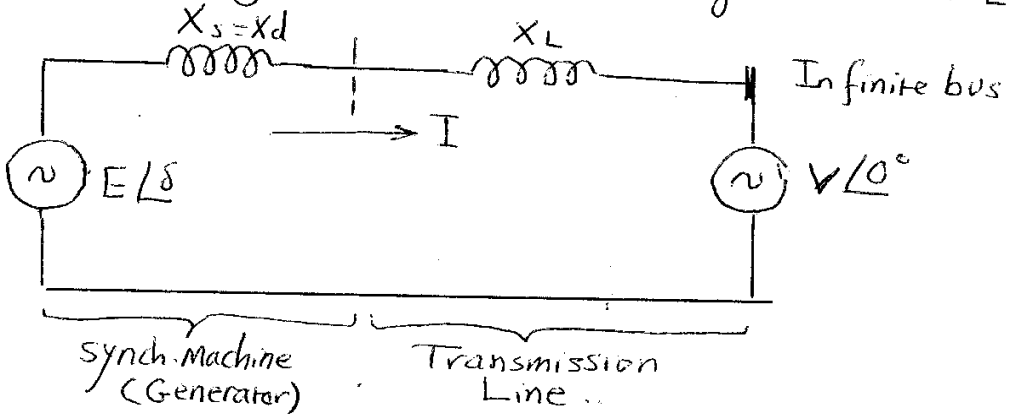
P_s is called the Synchronizing Power Coefficient and is equal to the slop of the power-angle curve.



- As long as P_s is positive, i.e $0° \leq \delta \leq 90°$, if $\delta \uparrow$, $P \uparrow$ and the generator is stable.
- Maximum power results when $\delta = 90°$, or:
  $$P_{max} = \frac{3|E||V|}{X_s}$$
- In practice $\delta \cong 30°$.

Transfer Reactance and Power Transfer

Consider a synchronous generator connected to an infinite bus through transmission line of reactance X_L.



The generator has synch. reactance X_s (or X_d) combines the effects of stator leakage reactance and armature reaction.

The complex power delivered by the generator to the infinite bus (system) is:

$$S = VI^* = |V|\left[\frac{|E|\angle\delta - |V|}{j(X_d + X_L)}\right]^*$$

$$= \frac{|E||V|}{X}\angle 90 - \delta \; - j\frac{|V|^2}{X}$$

$$= \frac{|E||V|}{X}\sin\delta + j\left(\frac{|E||V|}{X}\cos\delta - \frac{|V|^2}{X}\right)$$

where $X = X_d + X_L$.

The real power delivered to the system is

$$P_e = \mathrm{Re}[S] = \frac{|E||V|}{X}\sin\delta$$

$$= P_{max}\sin\delta \qquad \text{------- (1)}$$

where $P_{max} = \dfrac{|E||V|}{X}$. (Max. power limit).

- The total reactance X is called the Transfer Reactance and has important effect on power-angle curve.
- The maximum power limit is inversely proportional to the transfer reactance X.
- Equation (1) is valid for steady-state as well as transient conditions. For steady-state conditions we use synchronouse reactance X_s (or X_d) and E, while for transient condition X_d' and E' are used.

Example: Fig.1 shows a generator-transformer unit feeding an infinite bus through a transmission line. At the middle of the transmission line a 3-phase inductor of reactance 0.5 p.u. per phase is connected. The generator e.m.f behind synchronous reactance is 1.1 p.u. and the voltage of infinite bus is 1 p.u. Find:

(a) transfer reactance and steady-state power with inductor switch open;

(b) transfer reactance and steady-state power limit with inductor switch closed;

(c) transfer reactance and steady-state power limit with inductor replaced by capacitor of same per unit reactance.

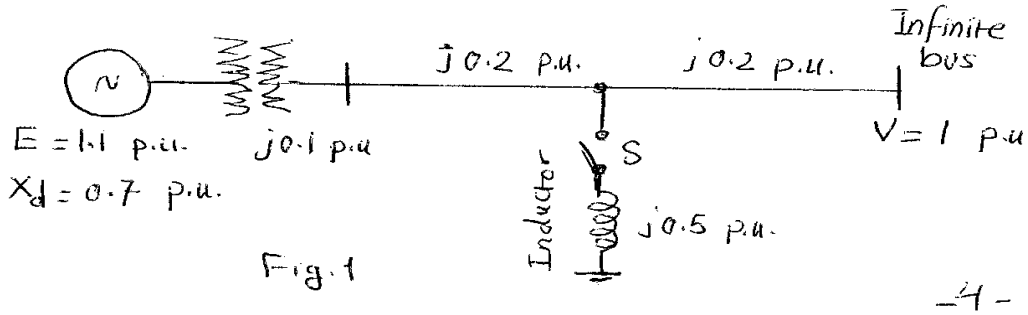


Fig.1

Solution :

(a) when switch S is open, the transfer reactance is:

$$X = 0.7 + 0.1 + 0.2 + 0.2 = 1.2 \text{ p.u.}$$

The steady-state power limit $= \frac{|E||V|}{X} = \frac{(1.1)(1)}{1.2} = 0.916$ p.u.

(b) when switch S is closed, the equivalent ccT will be as shown in Fig.1 (a) :

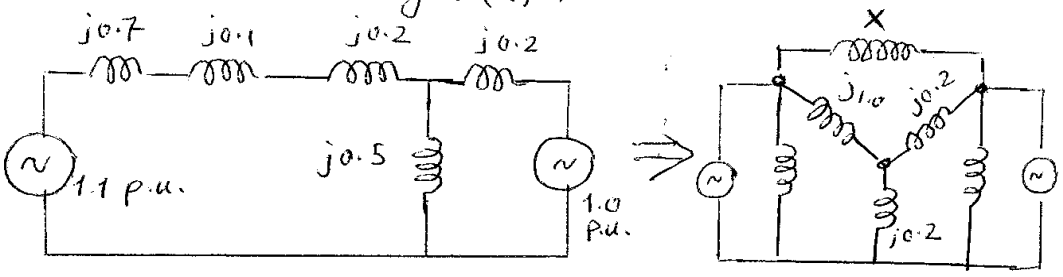
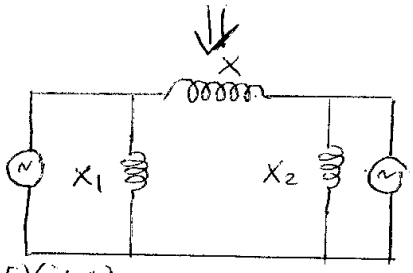


Fig.1 (a)

Converting the Star to delta.



$$X = \frac{(j1.0)(j0.2) + (j0.2)(j0.5) + (j0.5)(j1.0)}{j0.5}$$

$$= j1.6 \text{ p.u.}$$

which is the transfer reactance.

(X_1 and X_2 are not necessarily to be calculated since they do not contribute the power transfer.)

$$\text{Steady-state limit} = \frac{(1.1)(1)}{1.6} = 0.6875 \text{ p.u.}$$

(C) when the inductive reactor is replaced by a capacitor of 0.5 p.u. The configuration will be as in Fig.1(b):

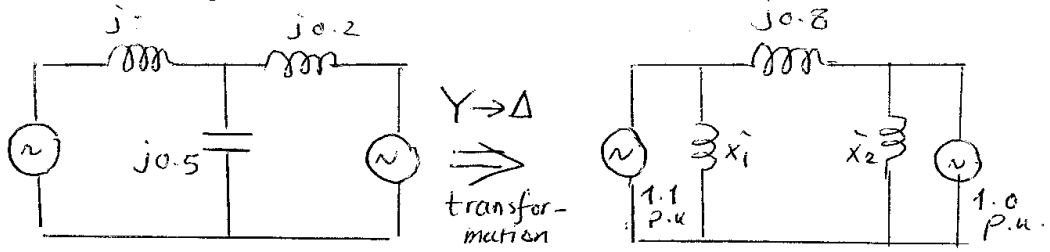


Fig.1(b)

The transfer reactance is obtained by changing the Y-Δ:

$$X = \frac{(j1.0)(j0.2) + (-j0.5)(j0.2) + (-j0.5)(j1.0)}{(-j0.5)}$$

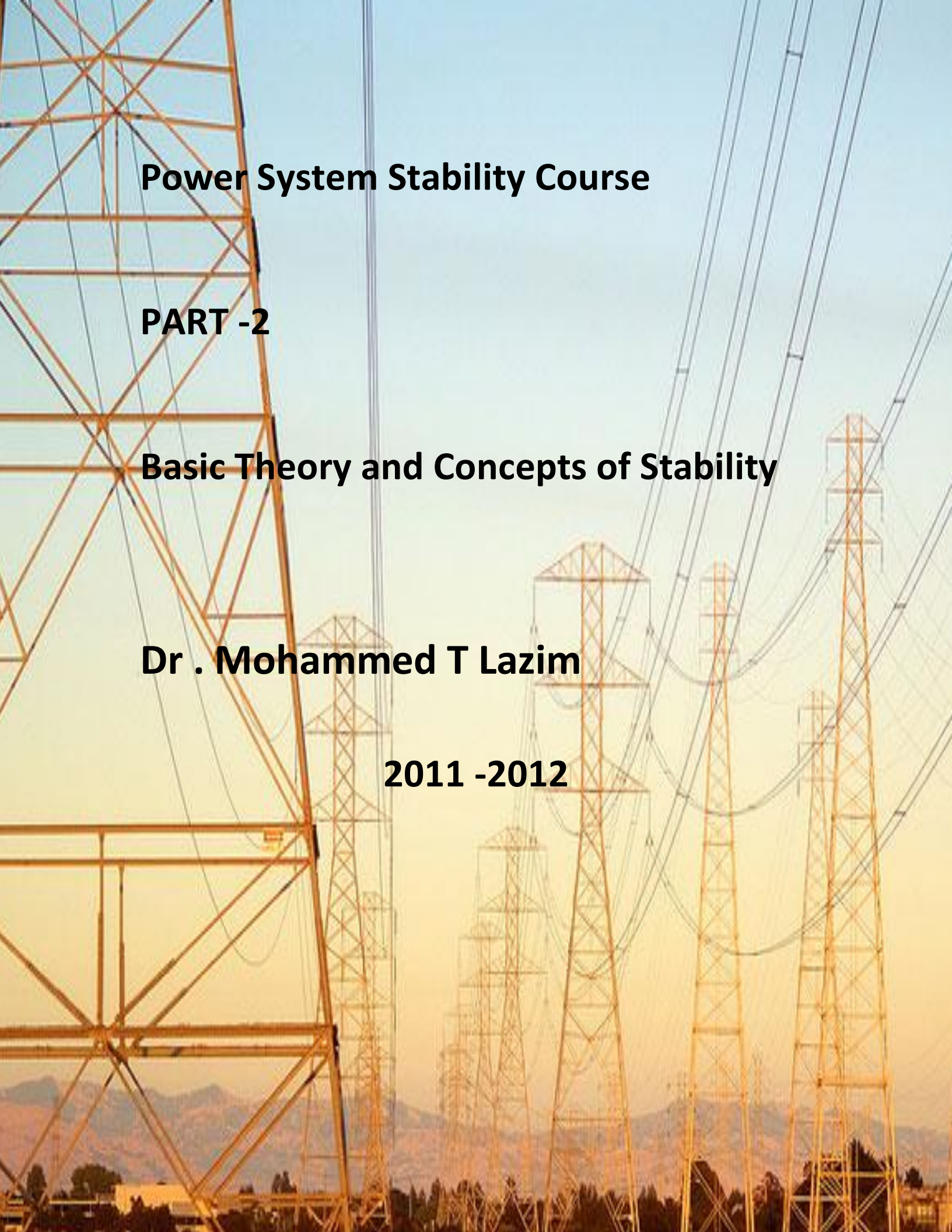
$$= j0.8 \text{ p.u.}$$

$$\text{Steady-state power limit} = \frac{(1.1)(1.0)}{0.8} = 1.373 \text{ p.u.}$$

H.W: Find the transfer reactance and steady-state power limit if instead of shunt capacitor in the previous example, a series capacitor of same per unit value reactance is connected in series with the line at same location. Sketch the circuit configuration.

Ans: $X = j0.7$ p.u.

S-S power limit = 1.57 p.u.



Power System Stability Course

PART -2

Basic Theory and Concepts of Stability

Dr . Mohammed T Lazim

2011 -2012

PART - 2

Stability of Power Systems Dr. Mohammed Tawfeeq

Review of Rotational Mechanics Laws

Rotational Mechanics -VS- Linear Mechanics

Rotational mechanics is very important in Stability Studies. The following table gives a comparison between the Linear and rotational mechanics formulea:

Rotational Mechanics	Linear Mechanics
θ = Angular displacement (rad.)	S = Displacement (distance) (m)
ω = Angular velocity (rad/sec) $= \frac{d\theta}{dt}$	v = velocity (m/sec.) $= \frac{ds}{dt}$
α = Angular accelaration (rad^*/sec^2) $= \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	a = accelaration (m/sec^2) $= \frac{dv}{dt} = \frac{d^2s}{dt^2}$
J = Moment of inertia ($Kg.m^2$)	m = Mass (Kg)
T = Torque (N.m) $= J\alpha$	F = Force (N) $= ma$.
M = Angular momentum ($Kg.m^2.rad/sec$) $= J\omega$	M = Momentum (Kg.m/sec) $= mv$
K.E $= \frac{1}{2} J\omega^2$ (Joules)	K.E. $= \frac{1}{2} m v^2$ (Joules) (Kinetic energy)
Power = P = T.ω (watt)	Work = F.S N.m
Work = $\int P dt = \int T \frac{d\theta}{dt} = T\theta$.	$P = \frac{d}{dt}(F.s) = F\frac{ds}{dt} = F.v$. N.m/sec = Watt.

Usefull Formulea

Relations between mechanical and electrical Angles (related to electrical machines):

1° mechanical $= \left(\frac{P}{2}\right)^\circ$ electrical

wher P = no. of poles.

ω (mechanical) $= 2\pi n$ mechanical. rad/sec

where n = speed in revolution/sec. (rps).

or $\omega = \frac{2\pi n}{60}$; n = speed in revolution/min (rpm)

ω (electrical) $\Longrightarrow$ Angular frequency

Here $\omega = 2\pi f$ electrical rad/sec.

$= 360 f$ electrical degree/sec.

1° electrical $= \left(\frac{2}{P}\right)^\circ$ mechanical.

one revolution $= \frac{P}{2} \times 360$ electrical degree

1° electrical $= \frac{2}{P} \times \frac{1}{360}$ revolution.

1° electrical/sec $= \frac{2}{P} \times \frac{1}{360}$ rev./sec.

1° electrical/min $= \frac{2}{P} \times \frac{1}{360} \Big/ \frac{1}{60} = \frac{2}{P} \times \frac{60}{30}$

$= \frac{1}{3P}$.

Dynamics of the Synchronous Machine and the Swing Equation

Consider a single machine consists of a prime-mover and generator as shown in Fig.1.

The mechanical relations of this system are as follows:

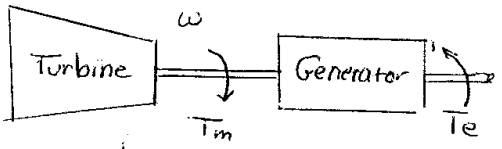


Fig.1

- The Kinetic energy of the rotor of a synchronous machine (generator) is given by:

$$K.E. = W_k = \frac{1}{2} J \omega^2 \quad \text{----(1)}$$

where:

J : the moment of inertia of the rotor and the prime-mover connected to it.

ω : the angular speed of the rotor.

- The angular momentum of the rotor is

$$M = J\omega \quad \text{------ (2)}$$

- From (1) and (2), the kinetic energy of the rotor of the synchronous generator can be written as:

$$K.E = \frac{1}{2} M \omega \quad \text{----(3)}$$

- Using eq(3), the angular momentum can also be expressed as:

$$M = \frac{2(K.E)}{\omega} \quad \text{-----(4)}$$

Inertia Constant H :

For dynamic studies a per unit quantity called the inertia constant H is used.
H is defined as :

$$H = \frac{\text{Stored energy in Mega Joules}}{\text{Machine rating in MVA}}$$

$$= \frac{K.E}{G} \qquad \text{------ (5)}$$

where G = Machine rating in MVA.

Hence eq.(5) may be written as :

$$\boxed{K.E = HG} \qquad \text{------ (6)}$$

or $\frac{1}{2}M\omega = HG = \frac{1}{2}M(360f)$ ---(7)

(since $\omega = 2\pi f$ and $2\pi = 360$).

From eq.(7) we can express M either in terms of electrical degree or in electrical radian as:

$$\boxed{M = \frac{GH}{180f}}$$ Mega-joules. Sec./elect. degree

or

$$\boxed{M = \frac{GH}{\pi f}}$$ Mega-joules. sec./electr. rad.

------ (8)

Table 1. gives typical values of H for different types of synchronous machines.

Table-1

Type of machine	H (MJ/MVA)
Turbo-generator	3.9
Water wheel generator	2.4
Synchronous condenser	1 - 1.25
Synchronous motor	2.0

The Swing Equation of a Synchronous generator

- Single-machine system

Consider a single generator shown as a block diagram in Fig.1

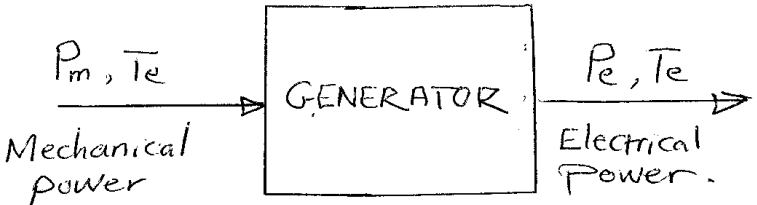
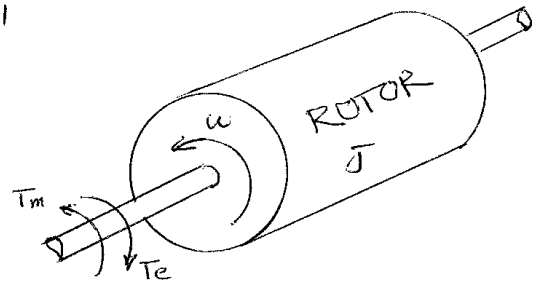


Fig.1



The Newton's second law, applied to the motion of the rotor of the synchronous generator has the form:

$$J\alpha = Ta \qquad \text{-------} \quad (9)$$

where α is the angular accelaration and

$Ta = Tm - Te$ is the accelarating torque.

J = Moment of inertia of the rotor ($Kg.m^2$)

Tm = Mechanical torque in N.m.

Te = Electrical torque in N.m.

but $\alpha = \frac{d^2\theta}{dt^2}$

Hence eq.(9) can be written as:

$$J\frac{d^2\theta}{dt^2} = Tm - Te \quad (N.m) \qquad \text{-------} \quad 10$$

The angle θ is measured with respect to a stationary reference frame on the stator. However, it is more convinient to measure the rotor angular position w.r.t a reference frame axis which rotates at synchronous speed, hence:

$$\theta = \omega_s t + \delta \qquad \text{------ (11)}$$

where:

ω_s = Synchronous speed of the rotor (rad/sec)

δ = the angular displacement of the rotor (radians)

Now differentiate eq.(11) w.r.t (t):

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt}$$

and $\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$ (since ω_s is constant). ------- (12)

Substitute eq.(12) in eq.(10) yield:

$$J\frac{d^2\delta}{dt^2} = T_a = T_m - T_e \qquad \text{------ (13)}$$

Multiplying both sides of eq.(13) by ω:

$$J\omega\frac{d^2\delta}{dt^2} = \omega T_a = \omega T_m - \omega T_e$$

or

$$\boxed{M\frac{d^2\delta}{dt^2} = P_a = P_m - P_e} \qquad \text{---- (14)}$$

where:

$M = J\omega$, $P_a = \omega T_a$ = accelarating power

$P_m = \omega T_m$ = mechanical power.

$P_e = \omega T_e$ = electrical power.

Equation (14) is called the <u>swing equation</u> of the rotor of the synch. generator

Electrical and Mechanical Form of the Swing Equation

We have derive the swing equation in the general form (eq.(14)) as:

$$M\frac{d^2\delta}{dt^2} = P_a = P_m - P_e$$

From eq.(8): $M = \frac{GH}{\pi f} = \frac{H}{\pi f} \times \frac{G}{G}$ in p.u.

or $M = \frac{2H}{2\pi f} \times \frac{G}{G} = \frac{2H}{\omega_s}$ in p.u.

but $\boxed{\omega_s = \frac{P}{2}\omega}$ where: ω = Mechanical angular velocity

Hence the swing equation can be written as:

$$\boxed{\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e}$$ Electrical Form

or

$$\boxed{\frac{2H}{\omega}\cdot\frac{2}{P}\cdot\frac{d^2\delta}{dt^2} = P_m - P_e}$$ Mechanical Form

Note: The swing equation is non-linear second order differential equation, this is because the electrical power generated by the synchronous generator (P_e) and thus the accelarating power P_a are non-linear function of the angle δ.

$$P_e = \frac{|E||V|}{X}\sin\delta \quad \text{or} \quad P_e = P_{max}\sin\delta.$$

Example 1: A 2-pole, 50 Hz, 60 MVA turbogenerator has a moment of inertia of 9×10^3 Kg.m^2. Calculate

(a) The Kienetic energy in MJ at rated speed

(b) The inertia constants H and M.

(c) The constant M on 50 MVA base.

Solution:

$$n = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm.}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi (3000)}{60} = 100\pi \text{ rad/sec.}$$

(a) K.E Stored $= \frac{1}{2} J \omega^2 = \frac{1}{2} \times 9 \times 10^3 (100\pi)^2 = 444 \times 10^6$ Joules

$= 444$ MJ

(b) $$H = \frac{\text{Kienetic energy stored in MJ}}{\text{MVA rating (G)}} = \frac{444}{60} = 7.4 \text{ MJ/MVA}$$

$$M = \frac{2H}{\omega_s} G = \frac{HG}{180f} = \frac{7.4 \times 60}{180 \times 50} = \frac{444}{180 \times 50} =$$

$= 0.0493$ MJs/elect. degree.

(c) Per unit M $= \frac{M}{\text{base MVA}} = \frac{0.0493}{50} = 9.86 \times 10^{-4}$ p.u.

Two-Machine System

Consider two machines connected to an infinite bus as shown in Fig.1. The swing equations on common system base are:

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \quad \text{P.U...(1)}$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m2} - P_{e2} \quad \text{P.U...(2)}$$

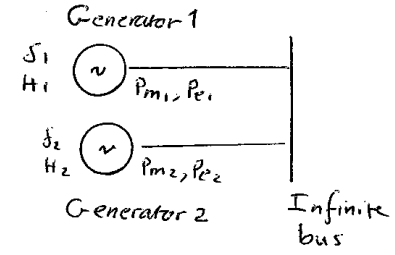


Fig.1

Now there are two possible cases:

1. Machies swing together ($\delta_1 = \delta_2 = \delta$) are called Coherent machines, hence:

$$\frac{2H_1}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m1} - P_{e1} \quad \text{.....(3)}$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m2} - P_{e2} \quad \text{....(4)}$$

Adding (3) & (4) yields:

$$\frac{2}{\omega_s}(H_1+H_2)\frac{d^2\delta}{dt^2} = (P_{m1}+P_{m2}) - (P_{e1}+P_{e2}) \quad \text{....(5)}$$

or $\frac{2}{\omega_s} H \frac{d^2\delta}{dt^2} = P_m - P_e$ $\quad H = H_1 + H_2$ (equivalent inertia)

2. Machines not swing together ($\delta_1 \neq \delta_2$) are called noncoherent machines, hence:

Subtracting (2) from (1) we obtain:

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left[\frac{P_{m1}-P_{e1}}{H_1} - \frac{P_{m2}-P_{e2}}{H_2}\right] \quad \text{...(6)}$$

multiplying both side of eq.(6) by H_1H_2/H_1+H_2 and rearrange:

$$\frac{2}{\omega_s}\left(\frac{H_1 H_2}{H_1+H_2}\right)\frac{d^2(\delta_1-\delta_2)}{dt^2} = \frac{P_{m_1}H_2 - P_{m_2}H_1}{H_1+H_2} - \frac{P_{e_1}H_2 - P_{e_2}H_1}{H_1+H_2}$$

in simple form:

$$\frac{2}{\omega_s} H_{12} \frac{d^2\delta_{12}}{dt^2} = P_{m_{12}} - P_{e_{12}} \quad \text{- - - - (8)}$$

the relative angle $\delta_{12} = \delta_1 - \delta_2$.

H_{12} = equivalent inertia $= \frac{H_1 H_2}{H_1+H_2}$ (both H_1 & H_2 must be on same base).

- Question: How to find equivalent H when H_1 & H_2 are given on different bases?

Example: Two generating units operate in parallel with the same power plant and have the following ratings:

Unit 1: 500 MVA, 0.85 p.f, 20 kV, 3500 rpm, 50 Hz
$H_1 = 4.8$ MJ/MVA

Unit 2: 750 MVA, 0.9 P.f. 20 kV, 1500 rpm, 50 Hz
$H_2 = 3.5$ MJ/MVA

Calculate equivalent H constant for the two machines on a 100-MVA base.

Solution: The total kinetic energy (KE) of rotation of the two machine is

$$KE = H_1 G_1 + H_2 G_2 = 4.8 \times 500 + 3.5 \times 750$$

$$= 2400 + 2625 = 5025 \text{ MJ}.$$

$$\equiv H_{eq} \cdot G_b$$

$$\therefore H_{eq} = \frac{KE}{G_b} = \frac{5025}{100} = 50.25 \text{ MJ/MVA}.$$

Equivalent M for TWO Machines:

For the two-machine system shown in Fig.1:

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} = \Delta P_1 \quad \text{- - - - - (9)}$$

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} = \Delta P_2 \quad \text{- - - - - (10)}$$

- For <u>Coherent Machines</u>: $\delta_1 = \delta_2 = \delta$:

Adding (9) & (10)

$$(M_1 + M_2) \frac{d^2\delta}{dt^2} = \Delta P_1 + \Delta P_2$$

or $$M_{eq} \frac{d^2\delta}{dt^2} = \Delta P$$

$$\therefore \boxed{M_{eq} = M_1 + M_2} \quad \text{- - - - - - - (11)}$$

- For <u>noncoherent machines</u> $\delta = \delta_1 - \delta_2$ (relative angle between the two machines):

The relative acceleration between the two machine is

$$\frac{d^2\delta}{dt^2} = \frac{d^2\delta_1}{dt^2} - \frac{d^2\delta}{dt^2} = \frac{\Delta P_1}{M_1} - \frac{\Delta P_2}{M_2}$$

but $\Delta P_2 = -\Delta P_1$ for power balance, hence

$$\frac{d^2\delta}{dt^2} = \frac{\Delta P_1}{M_1} - \frac{(-\Delta P_1)}{M_2} = \Delta P\left(\frac{1}{M_1} + \frac{1}{M_2}\right) = \frac{\Delta P}{M_{eq}} = \frac{\Delta P}{\frac{M_1 M_2}{M_1 + M_2}}$$

$$\therefore \boxed{M_{eq} = \frac{M_1 \cdot M_2}{M_1 + M_2}} \quad \text{- - - - - - - - (12)}$$

<u>Note</u>: The angular momentum, M, is not strictly constant, but it can be regarded constant as the swing equation is normally solved for short times (in the order of 1 sec.).

Multi-Machine System

For the purpose of stability studies, several machines at one generating station are usually considered as a single-machine. This equivalent machine is considered to have a rating equal to the sum of the ratings of the several machine considered to act together, during the transient Period and also to considered as coherent machines.

Hence for n-machine system:

- $G_{eq} = G_1 + G_2 + G_3 + \cdots + G_n$
- The moment of inertia of the rotor of the equivalent machine will be the sum of the moment of inertia for for these machines or:

$$\bar{J}_{eq} = \bar{J}_1 + \bar{J}_2 + \bar{J}_3 + \cdots + \bar{J}_n .$$

- This also applied to the angular momentum or:

$$M_{eq} = M_1 + M_2 + M_3 + \cdots + M_n .$$

- For the inertia constant H_{eq} can be found from:

$$K.E = G_{eq} \cdot H_{eq} = G_1 H_1 + G_2 H_2 + G_3 H_3 + \cdots + G_n H_n .$$

or $$H_{eq} = \frac{G_1}{G_{eq}} H_1 + \frac{G_2}{G_{eq}} H_2 + \frac{G_3}{G_{eq}} H_3 + \cdots + \frac{G_n}{G_{eq}} H_n .$$

Example2: A 200 MVA, 11 kV, 50 Hz, 4-pole turbo-generator has an inertia constant of 6 MJ/MVA.

(a) Find the stored energy in the rotor at synch. speed.

(b) The machine is operating at a load of 120 MW when the load is suddenly increases to 160 MW. Find the rotor retardation. Neglect losses.

(c) The retardation calculated in (b) above, is maintained for 5 cycles, find the change in power angle and rotor speed in r.p.m at the end of this period.

(d) Another generator 150 MVA, 300 r.p.m, having H = 4 MJ/MVA is put in parallel with the above generator. Find the equivalent inertia constant (H_{eq}) of the two generator on base of 100 MVA.

Solution

(a) Stored energy : K.E = GH = 200 X 6 = 1200 MJ.

(b) $P_a = 120 - 160 = -40$ MW ($P_a = P_m - P_e$)

$$M = \frac{GH}{180f} = \frac{1200}{180 \times 50} = \frac{2}{15} \text{ MJ.sec./elect.degree.}$$

but $M\alpha = P_a$

$$\frac{2}{15}\alpha = -40 \quad \therefore \alpha = -300 \text{ elect.deg./sec}^2.$$

Hence α is negative, it is a retardation and not accelaration, thus:

The retardation is 300 elect.deg./sec^2.

(c) The period of 5 cycles at 50 Hz is:

$5 \times 20\ ms = 100\ ms = 0.1$ second.

The change in angle δ can be found from:

$$\frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt} = \alpha$$

$$\int \frac{d^2\delta}{dt^2} = \int \alpha$$

$$\therefore \int d^2\delta = \int \alpha\, t\, dt$$

(since $\int dx^2 = \int x\, dx = \frac{1}{2}x^2$; $dx^2 = x\, dx$)

or $d\delta = \frac{1}{2}t^2\alpha$ (change in δ)

$$= \frac{1}{2}(0.1)^2(-300)$$

$$= -1.5 \text{ elect. deg.}$$

Change in rotor speed:

$\alpha = -300$ elect. deg./sec^2.

$$= (-300) \times \frac{1}{3P} = (-300) \times \frac{1}{3 \times 4} = -25 \text{ rpm.}$$

Note: To change elect. deg./sec^2 to rpm multiply by $(\frac{1}{3P})$. (From table on page 1)

n_s = Synch. speed of the rotor $= \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$ rpm.

$\therefore$ Rotor speed at the end of 5 cycles:

$$n_r = n_s + \alpha_{rpm} \cdot t = 1500 + (-25)(0.1)$$

$$= 1497.5 \text{ rpm.}$$

(d) K.E. $= H_1G_1 + H_2G_2 = H_{eq}G_{eq} = H_{eq} \cdot G_b$.

$$\therefore H_{eq} = \frac{H_1G_1}{G_b} + \frac{H_2G_2}{G_b} = \frac{6 \times 200}{100} + \frac{4 \times 150}{100}$$

$$= 18 \text{ MJ/MVA.}$$

Example 3: A 60Hz generator having $H = 6$ MJ/MVA is connected to a synchronous motor having $H = 4$ MJ/MVA through a network of reactances. The generator is delivering power of 1.0 p.u to the motor when a fault occurs which reduces the delivered power. At the time when the reduced power delivered is 0.6 p.u. determine the angular accelaration of the generator with respect to the motor.

Solution:-

After disturbance, the two machines becomes noncoherent. Hence the equation govern both machine is:

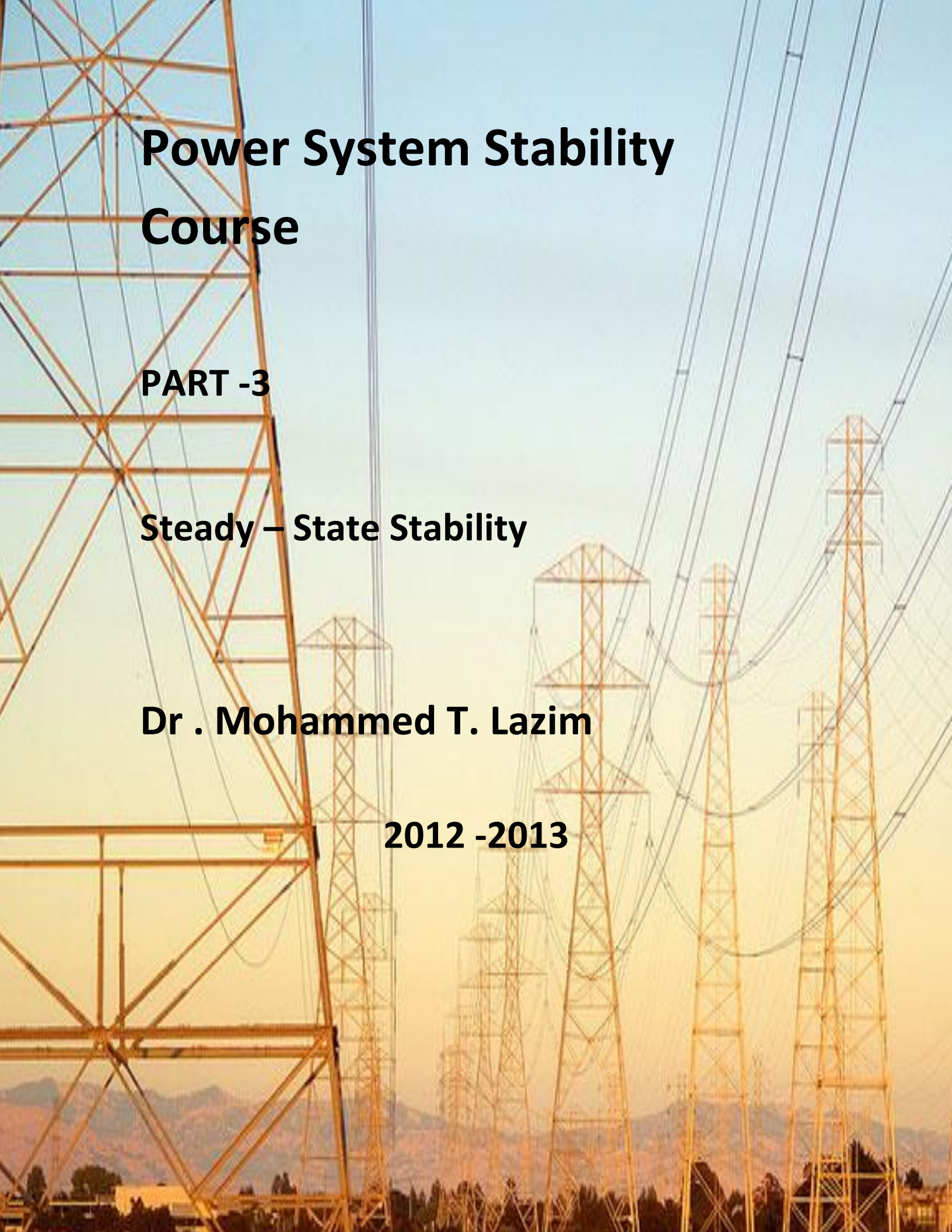
$$\frac{2}{\omega_s} H_{12} \frac{d^2\delta_{12}}{dt^2} = P_{m_{12}} - P_{e12}$$

$$H_{12} = H_{eq} = \frac{H_1 H_2}{H_1 + H_2} = \frac{4 \times 6}{4+6} = 2.4 \text{ MJ/MVA}$$

$$\frac{2}{2\pi f} \times 2.4 \frac{d^2\delta_{12}}{dt^2} = 1.0 - 0.6$$

for $f = 60$ Hz

$$\frac{2.4}{180 \times 60} \frac{d^2\delta_{12}}{dt^2} = 0.4 \qquad \therefore \frac{d^2\delta_{12}}{dt^2} = 1800 \text{ elec. deg./sec}^2.$$



Power System Stability Course

PART -3

Steady – State Stability

Dr . Mohammed T. Lazim

2012 -2013

Power System Stability — Dr.M.T.Lazim

Part 3 : STEADY-STATE STABILITY

- The steady-state stability was defined previously as: the ability of the system to remain in synchronizm when subjected to small disturbances.
- These small disturbances are caused by slow changes in load or input power to the generator.

- To study the steady-state stability problem, the following assumps are made:
  - * The disturbances causing the changes disappear, the motion of the system is free.
  - * The equations govern the behaviour of the system is linear.
  - * The automatic control devices, such as the voltage regulator and the speed governer are not active.

Consider a single machine connected to an infinite bus bar as shown in Fig.1

The swing equation was give in electrical form as:

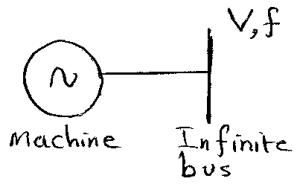


Fig.1

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

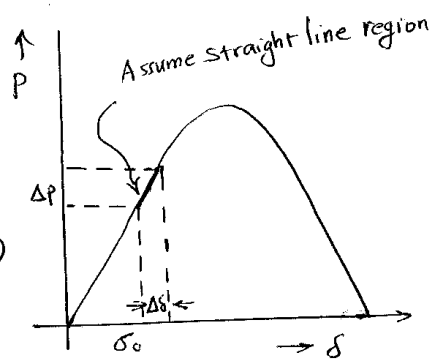
but $\omega_s = 2\pi f$, hence

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e = P_m - P_{max}\sin\delta \quad ----(1)$$

This is a non-linear D.E w.r.t δ.

However for small disturbances, the swing equation may be linearized as follows:

Consider a small deviation $\Delta\delta$ in power angle from initial operating angle δ_0 due to increase in the mechanical power ΔP, hence:



$$\delta = \delta_0 + \Delta\delta \quad \text{-----(2)}$$

Substitute eq(2) into (1) we get

$$\frac{H}{\pi f_s}\frac{d^2(\delta_0+\Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0+\Delta\delta) \quad \text{----(3)}$$

$$\frac{H}{\pi f_s}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}[\sin\delta_0\cos\Delta\delta + \cos\delta_0\sin\Delta\delta]$$

Since $\Delta\delta$ is small, $\cos\Delta\delta \cong 1$, and $\sin\Delta\delta \cong \Delta\delta$.

Thus:

$$\frac{H}{\pi f_s}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}\sin\delta_0 - P_{max}\cos\delta_0\,\Delta\delta$$

Since at the operating state,

$$\frac{H}{\pi f_s}\frac{d^2\delta_0}{dt^2} = P_m - P_{max}\sin\delta_0$$

$$\cancel{P_m} - \cancel{P_{max}\sin\delta_0} + \frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} = \cancel{P_m} - \cancel{P_{max}\sin\delta_0} - P_{max}\cos\delta_0\,\Delta\delta$$

$$\therefore \frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0\,\Delta\delta = 0 \quad \text{------(4)}$$

This is a linear D.E w.r.t $\Delta\delta$.

We have previously define $P_{max}\cos\delta_0 = P_s$ as the synchronizing power coefficient. Therefore substitute this value into eq.(4) we get:

$$\frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} + P_s\,\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_s P_s}{H}\Delta\delta = 0 \quad . \qquad \equiv \left(\frac{d^2x}{dt^2} + ax = 0 \quad S.H.M\right)$$

Using Laplace transform :

$$\Delta\delta\left(s^2 + \frac{\pi f_s P_s}{H}\right) = 0 \quad .$$

Solution of this equation yield :

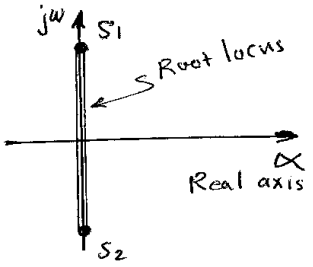
$$s^2 + \frac{\pi f_s P_s}{H} = 0 \quad . \qquad s = \pm\sqrt{\frac{-\pi f_s P_s}{H}} = \pm\sqrt{\frac{-P_s}{M}}$$

Since P_s is the slop of the power-angle curve, it can be either positive or negative. Hence stability assesment may be done as follows :

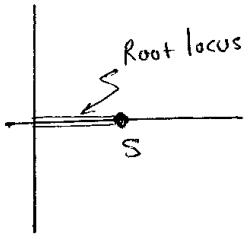
1. When P_s is positive, both roots $s_1 = +\sqrt{\frac{-P_s}{M}}$ and $s_2 = -\sqrt{\frac{-P_s}{M}}$ are on the jw axis (imajinary) and the motion is oscillatory and undamped with natural frequency of :

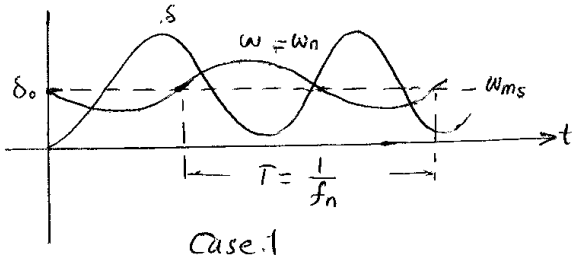
$$\omega_n = \sqrt{\frac{\pi f_s}{H} P_s} = \sqrt{\frac{P_s}{M}} \quad \text{rad/sec}$$

(neglecting losses and damping effects)

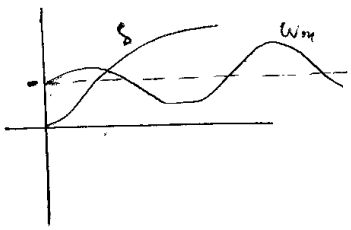


2. When P_s is negative, there will be on root in the right-half of s-plane and the response is exponentially increasing and Stability is lost (unstable case), this means that δ is increased without bound and lose of synchronizm occur.





Case.1



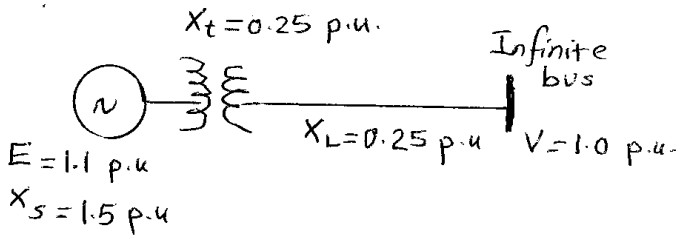
Case.2

sss/3

Examples on Steady-State Stability

Example 1 : A synchronous generator of reactance 1.5 p.u. is connected to an infinite bus system ($V = 1.0$ p.u.), through a line and transformers of total resistance 0.5 p.u. The no load voltage of the generator is 1.1 p.u. and the inertia constant $H = 5$ MW-s/MVA. All per unit values are expressed on the same base, resistance and damping effects may be neglected. Calculate the frequency of the oscillation set up when the generator operates at a load angle of 60° and is subjected to small disturbance. The system frequency is 50 Hz.

Solution



P_s = Synchronizing Coefficient

$$= \left.\frac{dP}{d\delta}\right|_{\delta=\delta_0} = \frac{|E||V|}{X_s}\cos\delta = \frac{(1.1)(1.0)}{2}\cos 60^\circ$$

$$= 0.272 \text{ p.u.}$$

$$S_1 \text{ and } S_2 = \pm j\sqrt{\frac{-P_s}{M}} = \pm\sqrt{\frac{-0.275}{5\times\left(\frac{1}{\pi\times 50}\right)}} = \pm j\sqrt{8.64}$$

The frequency of oscillation:

$\omega_n = 2.94$ rad/sec $\quad\therefore f_n = \frac{2.94}{2\pi}$ Hz.

or $f_n = 0.468$ Hz.

and the periodic time $= \frac{1}{f_n} = \frac{1}{0.468} = 2.14$ Sec.

Example 2 : A synchronous generator of reactance 1.20 p.u. is connected to an infinite bus bus ($V = 1.0$ p.u.) through transformers and and a line of total reactance 0.60 p.u. The generator no load voltage is 1.20 p.u. and its inertia constant is $H = 4$ MW.s/MVA. The resistance and machine damping may be neglected. The system frequency of normal oscillation is 50Hz. Calculate the frequency of natural oscillation if the generator is loaded:

(a) 50% and

(b) 80%

of its maximum power limit.

Solution on next page ⟹

Solution of Example 2

(a) For 50% Loading

$$\sin \delta_0 = \frac{P_e}{P_m} = 0.5 \quad \text{or} \quad \delta_0 = 30^\circ$$

$$P_s = \left.\frac{dP_e}{d\delta}\right|_{30^\circ} = \frac{|E||V|}{X} \cos \delta_0$$

$$= \frac{1.2 \times 1}{1.2 + 0.6} \cos 30 = 0.577 \quad \text{MW (P.u)/elect. rad}$$

$$M = \frac{H}{\pi f} = \frac{4}{\pi \times 50} \quad \text{MW(P.u)} \cdot \text{sec}^2/\text{elect. rad}$$

From the characteristic equation

$$S_1, S_2 = \pm j\sqrt{\frac{P_s}{M}} = \pm j\sqrt{\frac{0.577}{\frac{4}{50\pi}}} = \pm j\, 4.76.$$

but $\omega_n = \sqrt{\frac{P_s}{M}} = 4.76$ rad/sec

$$= \frac{4.76}{2\pi} = 0.758 \text{ Hz}$$

(b) For 60% loading

$$\sin \delta_0 = \frac{P}{P_m} = 0.6 \quad \text{or} \quad \delta_0 = 53.1^\circ$$

$$P_s = \frac{1.2 \times 1}{1.8} \cos 53.1 = 0.4 \quad \text{MW (P.u)/elect. rad}$$

$$S_1, S_2 = \pm\sqrt{\frac{P_s}{M}} = \sqrt{\frac{0.4}{\frac{4}{50\pi}}} = \pm j\, 3.96$$

∴ The natural frequency of oscillation:

$$\omega_n = 3.96 \quad \text{rad/sec}$$

$$= \frac{3.96}{2\pi} = 0.63 \text{ Hz}.$$

Steady - State Stability Analysis Considering Damping Effects

During small disturbance the speed of the rotor $\omega_r \neq \omega_{(stator\ field)}$, hence induction motor action will take place between rotor and rotating synch. field results in a retarding or damping torque tending to minimize the difference between ω_r and ω_s. The damping power resulting from the damping torque is approximately proportional to the speed deviation:

$$P_D \propto \frac{d\delta}{dt} \quad \text{or} \quad P_D = D\frac{d\delta}{dt} \simeq D\frac{d\Delta\delta}{dt} \quad \text{---(6)}$$

If damping is accounted for, the linearized swing equation becomes: $\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a - P_D$

$$\frac{H}{\pi f_s}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\Delta\delta = 0 \quad \text{---------(7)}$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_s D}{H}\frac{d\Delta\delta}{dt} + \frac{\pi f_s P_s}{H}\Delta\delta = 0$$

In terms of second order differential equation:

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0 \quad \text{----(8)}$$

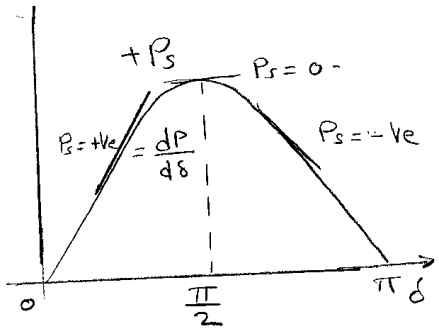
where

ω_n = natural frequency of oscillation

$$= \sqrt{\frac{\pi f_s P_s}{H}}$$

ζ = Damping ratio

$$= \frac{D}{2}\sqrt{\frac{\pi f_s}{H P_s}}$$



The characteristic equation

By transforming eq.(8) from time domain to frequency domain we get the characteristic equation:

$$S^2 + 2\zeta\omega_n S + \omega_n^2 = 0$$

Solving this equation using the formula $X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$:

If $\zeta < 1$, for normal operating conditions, we shall have complex roots:

$$S_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$
$$S_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

Let $\omega_d = \omega_n\sqrt{1-\zeta^2}$ The damping frequency of oscillation.

Laplace Transform Analysis

In order to extend the analysis to multi-machine system, we can convert the 2nd order D.E. (eq.(8)) into state-variable form as follows:

Let $\Delta\delta = X_1$

$\dot{\Delta\delta} = X_2 = \Delta\omega$ [The two variables are $\Delta\delta$ & $\Delta\omega$]

$\therefore \dot{X}_1 = X_2$ and $\ddot{X}_1 = \dot{X}_2$

or $\dot{X}_2 = -\omega_n^2 X_1 - 2\zeta\omega_n X_2$ [From eq.(8)]

In matrix form:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \text{---(9)}$$

or $\dot{\mathbf{X}} = A\mathbf{X}(t)$ State-variable equation

where $A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$

Taking Laplace transform, we have:

$$\mathcal{L}\{\dot{X}(t) = AX(t)\} \Rightarrow SX(s) - X(0) = AX(s)$$

$$SX(s) - AX(s) = X(0)$$

$$X(s)[SI - A] = X(0)$$

$$\therefore X(s) = [SI - A]^{-1} X(0)$$

But $[SI - A] = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$

where I = Unit matrix $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Thus $[SI-A] = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$

$$= \begin{bmatrix} S & -1 \\ \omega_n^2 & S+2\xi\omega_n \end{bmatrix}$$

&

$$[SI-A]^{-1} = \frac{\begin{bmatrix} S+2\xi\omega_n & 1 \\ -\omega_n^2 & S \end{bmatrix}}{\left| S^2+2\xi\omega_n S+\omega_n^2 \right|} \leftarrow \text{Determinant}$$

Hence

$$X(s) = \frac{\begin{bmatrix} S+2\xi\omega_n & 1 \\ -\omega_n^2 & S \end{bmatrix}}{\left| s^2+2\xi\omega_n s+\omega_n \right|} X(0) \quad \text{---(10)}$$

- When the rotor is suddenly disturbed by a small angle $\Delta\delta_0$, $X_1(0) = \Delta\delta_0$ and $X_2(0) = \Delta\omega_0 = 0$ (since there will be no sudden change in rotor speed), we therefore obtain:

$$X(s) = \begin{bmatrix} \Delta\delta(s) \\ \Delta\omega(s) \end{bmatrix} \quad , \quad X(0) = \begin{bmatrix} \Delta\delta_0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \Delta\delta(s) \\ \Delta\omega(s) \end{bmatrix} = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \begin{bmatrix} s + 2\xi\omega_n & 1 \\ -\omega_n & s \end{bmatrix} \begin{bmatrix} \Delta\delta_0 \\ 0 \end{bmatrix}$$

$$\text{or} \quad \Delta\delta(s) = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \left[(s + 2\xi\omega_n)\Delta\delta_0 + 1 * 0 \right]$$

$$\Delta\delta(s) = \frac{(s + 2\xi\omega_n)\Delta\delta_0}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{- - - - - - - (11)}$$

$$\& \quad \Delta\omega(s) = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \left[(-\omega_n^2)\Delta\delta_0 + s * 0 \right]$$

$$\Delta\omega(s) = \frac{-\omega_n^2 \Delta\delta_0}{s^2 + 2\xi\omega_n + \omega_n^2} \quad \text{- - - - - - - (12)}$$

Taking inverse Laplace transform of eqs (11) & (12):

$$\Delta\delta(t) = \frac{\Delta\delta_0}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \theta) \quad \text{- - - 13}$$

$$\Delta\omega(t) = \frac{-\omega_n \Delta\delta_0}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t) \quad \text{- - - - (14)}$$

where ω_d is the damped frequency of oscillation, and θ is give by:

$$\theta = \cos^{-1}\xi \quad \text{- - - - - - - - - - (15)}$$

- The motion of the rotor relative to synchronously rotating field is

$$\delta(t) = \delta_0 + \Delta\delta(t) \quad \text{- - - - - - (16)}$$

$$\tau = \frac{1}{\zeta \omega_n}$$

The settling time t_s is approximate four time-constant, i.e

$$t_s = 4\tau$$

Note: * As the inertia constant H increase, ω_n and ζ (damping ratio) are decrease resulting in long settling time t_s.

* As P_s (synch. power coefficient) increases, this results in an increase in ω_n and decrease in the damping ratio D.

* P_s has no effect on the settling time t_s (why?).

Answer: Since $t_s = 4\tau$ and $\tau = \frac{1}{\zeta \omega_n}$.

but $\omega_n = \sqrt{\frac{\pi f_s P_s}{H}}$ and $\zeta = \frac{D}{2}\sqrt{\frac{\pi f_s}{H P_s}}$

$\therefore \quad \omega_n \cdot \zeta = \frac{D}{2}\sqrt{\frac{\pi f_s}{H} P_s \cdot \frac{\pi f_s}{H P_s}} = \frac{D}{2}\left(\frac{\pi f_s}{H}\right)$.

or $\quad \tau = \frac{1}{\omega_n \zeta} = \frac{2H}{\pi f_s D}$ & $t_s = \frac{8H}{\pi f_s D}$.

SSS/11

Example 3: A 60 Hz synchronous generator having inertia constant $H = 9.94$ MJ/MVA and a transient reactance $X_d' = 0.3$ per unit is connected to an infinite bus through a pure reactive circuit as shown in Fig. 1. Reactances are marked on the diagram on a common system base. The generator is delivering real power of 0.6 p.u, 0.8 power factor lagging to the infinite bus at a voltage of $V = 1$ p.u. Assume the per unit damping power coefficient is $D = 0.138$. Consider a small disturbance of $\Delta\delta = 1.0^\circ = 0.1745$ radian. The breakers open and then close. Obtain equations describing the motion of the rotor angle and the generator frequency.

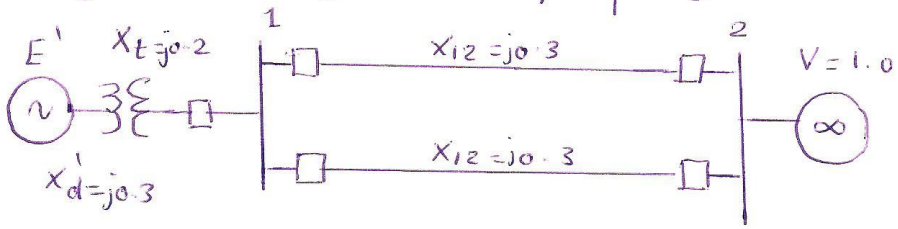


Fig. 1

Solution:

The transfer reactance between the generated voltage and the infinite bus is

$$X = j0.3 + j0.2 + j\frac{0.3}{2} = j0.65 \text{ p.u.}$$

The p.u. apparent power is

$$S = \frac{0.6}{0.8}\angle \cos^{-1} 0.8 = 0.75 \angle 36.87^\circ$$

The current is

$$I = \frac{S^*}{V^*} = \frac{0.75\angle -36.87^\circ}{1.0\angle 0^\circ} = 0.75\angle -36.87^\circ$$

The excitation voltage is

$$E' = V + jXI = 1.0\angle 0^\circ + j(0.65)(0.75\angle -36.87^\circ)$$
$$= 1.35\angle 16.79^\circ$$

SSS/12

Thus the initial operating power angle is $\delta_0 = 16.79^\circ$ or 0.2931 rad.

The synchronizing power coefficient is:

$$P_s = P_{max}\cos\delta_0 = \frac{(1.35)(1)}{0.65}\cos 16.79^\circ = 1.9884.$$

The undamped angular frequency of oscillation and damping ratio are:

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} = \sqrt{\frac{60\pi}{9.94}\times 1.9884} = 6.1405 \text{ rad/sec.}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_s}} = \frac{0.138}{2}\sqrt{\frac{60\pi}{9.94\times 1.9884}} = 0.2131$$

The linearized force-free equation which determines the mode of oscillation which is given by the equation:

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0.$$

or $$\frac{d^2\Delta\delta}{dt} + 2.62\frac{d\Delta\delta}{dt} + 37.7\Delta\delta = 0.$$

The damped angular frequency of oscillation is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.1405\sqrt{1-(0.2131)^2} = 6.0 \text{ rad/sec.}$$

and $$f_d = \frac{\omega_d}{\pi} = \frac{6.0}{2\pi} = 0.954 \text{ Hz}.$$

We have, the motion of the rotor relative to the synchronously revolving field is:

$$\delta = \delta_0 + \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t + \theta) \quad \Big| \theta = \cos^{-1}\zeta$$

$$= 16.79 + \frac{10}{\sqrt{1-(0.2131)^2}}e^{-(0.2131)(6.1405)t}\sin(6.0t + 77.69)$$

(10 ← degrees)

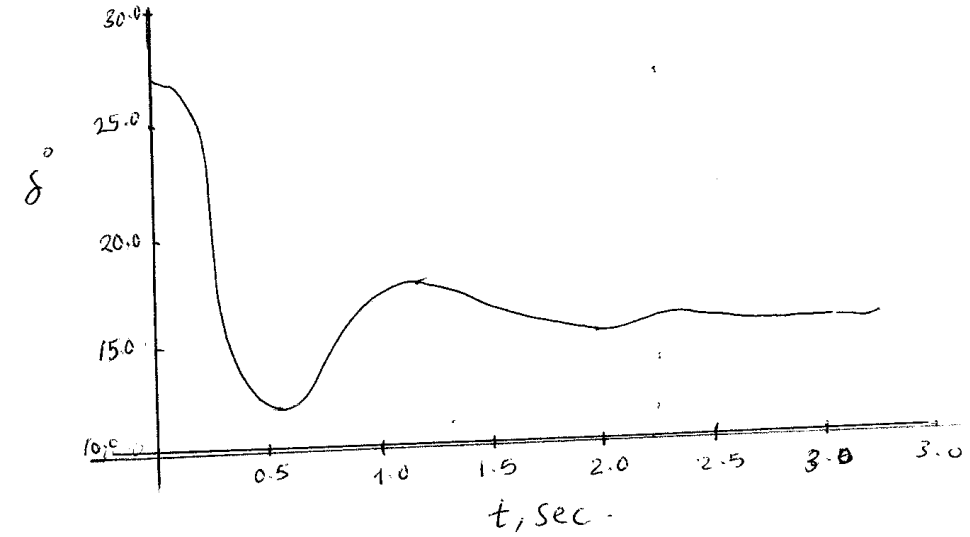
The rotor frequency f:

$$f = \frac{\omega}{2\pi} = \frac{\omega_0}{2\pi} - \frac{\omega_n\Delta\delta_0}{2\pi\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t$$

($\Delta\delta_0$ rad = 0.1745)

$$= 60 - 0.1746\,e^{-1.3t}\sin 6.0t. \qquad \text{SSS/13}$$

& $\delta = 16.79 + 10.23\,e^{-1.3t}(\sin(6t + 77.69^\circ))$



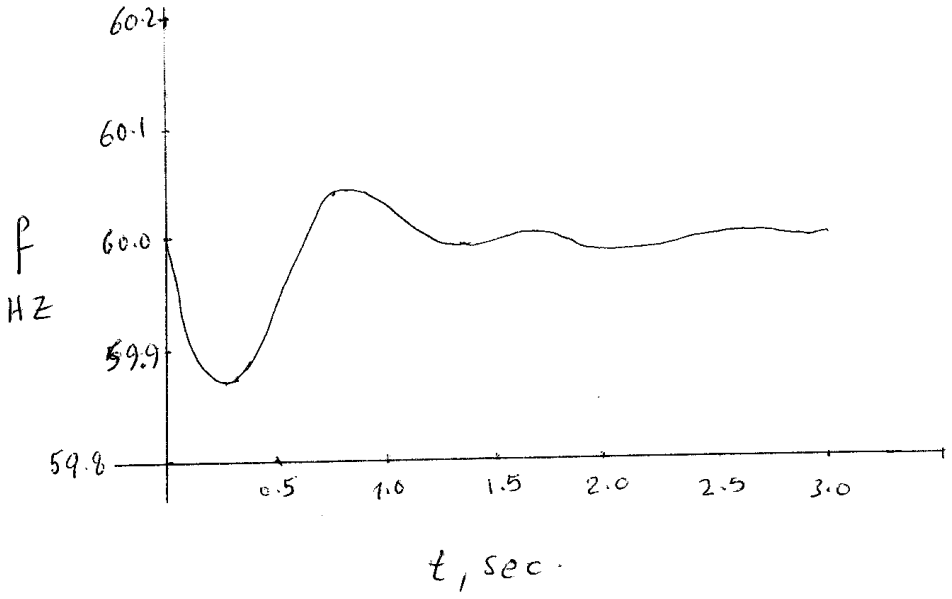


Fig. Natural response of the rotor and frequency for machine of the example-3

Steady-State Stability / Small Disturbance

System Response to small power Impact [Non-zero response]

Assume the power input is increased by ΔP (small amount of power). Then the linearized swing equation becomes

$$\frac{H}{\pi f_o}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\Delta\delta = \Delta P \qquad ----(1)$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_o}{H} D\frac{d\Delta\delta}{dt} + \frac{\pi f_o}{H} P_s \Delta\delta = \frac{\pi f_o}{H}\Delta P.$$

or in terms of the standard second-order differential equation, we have

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = \Delta u. \qquad ----(2)$$

where $\Delta u = \frac{\pi f_o}{H}\Delta P.$ ----- (3)

and $\zeta = \frac{D}{2}\sqrt{\frac{\pi f_o}{H P_s}}$ & $\omega_n = \sqrt{\frac{\pi f_o}{H} P_s}$.

Transforming to state variable form, we have

$$x_1 = \Delta\delta$$
$$x_2 = \dot{\Delta\delta} = \Delta\omega$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2 + \Delta u$$

In matrix form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\zeta\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\Delta u. \qquad ---(4)$$

or

$$\dot{X}(t) = AX(t) + B\Delta u(t). \qquad ------(5)$$

This is the forced state variable equation or the non-zero-state equation, x_1 and x_2 are the desired response.

SSS/15

Taking Laplace transform of the state equation (5) with zero initial state yields:

$$sX(s) = AX(s) + B\Delta U(s) \qquad X(0) = 0$$

or $$X(s) = (sI - A)^{-1} B \Delta U(s)$$

where $\Delta U(s) = \frac{\Delta u}{s}$ (step change).

Substituting for $(sI - A)^{-1}$, we get

$$X(s) = \frac{\begin{bmatrix} s+2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix} \begin{bmatrix} \omega \\ 1 \end{bmatrix} \frac{\Delta u}{s}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

or $$\Delta\delta(s) = \frac{\Delta u}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \qquad \text{------(6)}$$

and $$\Delta\omega(s) = \frac{\Delta u}{s^2 + 2\zeta\omega_n + \omega_n^2} \qquad \text{-------(7)}$$

Taking Laplace inverse transform results for step response:

$$\Delta\delta = \frac{\Delta u}{\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)\right] \qquad \text{---(8)}$$

where $\theta = \cos^{-1}\zeta$ and

$$\Delta\omega = \frac{\Delta u}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t \qquad \text{---(9)}$$

Substituting for Δu, the motion of the rotor relative to the synchronously rotating field, in electrical radian:

$\delta = \delta_o + \Delta\delta$

$$\delta = \delta_o + \frac{\pi f_o}{H\omega_n^2}\Delta P\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)\right] \qquad \text{--(10)}$$

and the rotor angular frequency in radian per sec. is

$\omega = \omega_o + \Delta\omega$

$$\omega = \omega_o + \frac{\pi f_o \Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t. \qquad \text{---(11)}$$

SSS/16

Example 4: For the previous example 3, the generator is operating in the steady-state at $\delta_o = 16.79^\circ$ when the input power is increased by a small amount $\Delta P = 0.2$ p.u. The generator excitation and the infinite busbar voltage are the same as before, i.e $E' = 1.35$ p.u and $V = 1.0$ p.u.

Using equations (10) and (11), obtain the step response for the rotor angle and the generator frequency.

Solution: Substitute for H, δ_o, ζ and ω_n evaluated in example 3, in eq.(10) and expressing the power angle in degree, we get:

$$\delta = 16.79^\circ + \frac{(180)(60)(0.2)}{(9.94)(6.1405)^2}\left[1 - \frac{1}{\sqrt{1-(0.2131)^2}} e^{-1.3t} \sin(6t + 77.7^\circ)\right]$$

$$\delta = 16.79^\circ + 5.7631\left[1 - 1.023 e^{-1.3t} \sin(6t + 77.7^\circ)\right]$$

Also subst. in eq.(11)

$$f = 60 + \frac{(60)(0.2)}{2(9.94)(6.1405)\sqrt{1-(0.2131)^2}} e^{-1.3t} \sin 6t.$$

or $f = 60 + 0.10\, e^{-1.3t} \sin 6t$.